Math 1A: Discussion 9/28/2018 Problems

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September 28, 2018

Problem Set 1

Question 1

Find the equation of the tangent line and the normal line to the graph of

$$f(x) = x^4 + x + e^x$$

at $x = 0$.

Problem Set 2

Question 2

Use the fact that

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

and the definition of the derivative to find the derivative of the following functions

$$y = e^{-2x} + x$$

$$y = 3^x - 3$$

Question 3

Is there a point $x = a$ such that the tangent line to $f(x) = \frac{1}{2}e^{2x}$ and $g(x) = 5e^x - 4x$ at $x = a$ have the same slope?

Problem Set 3

Question 4 (*)

Consider the function

$$f(x) = ae^{x-1} + 2b\sqrt{x} \text{ for } x \geq 1$$

$$f(x) = bx^3 + 2 \text{ for } x < 1$$

What values of a and b make the function $f(x)$ differentiable at $x = 1$? (Hint: Start by making $f(x)$ continuous at $x = 1$, because for $f(x)$ to be differentiable at $x = 1$, it needs to be continuous at $x = 1$ first)

Question 5 (*)

Note that we can iterate the process of taking derivatives. Since $f'(x)$ is a function, we can take its derivative to get what is called the second derivative $f''(x)$.

- What is a function $f(x)$ such that $f'(x) = f(x)$?
- Find two functions $f(x)$ that are not multiples of each other such that $f''(x) = f(x)$.
- Find two functions $f(x)$ that are not multiples of each other such that

  $$f''(x) - f'(x) - 6f(x) = 0$$

- Explain a general process you can use to find a function $f(x)$ such that

  $$f''(x) + af'(x) + bf(x) = 0$$

  Does your process always work? It probably doesn't - to see this, try to use it to find a solution to the equation

  $$f''(x) + f(x) = 0$$

  What goes wrong?