Math 1A: Discussion 9/26/2018 Solutions

Jeffrey Kuan

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Question 1

$$\lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{x+3} = \lim_{x\to\infty} \frac{\frac{\sqrt{4x^2+1}}{x}}{\frac{x+3}{x}} = \lim_{x\to\infty} \frac{\sqrt{\frac{4x^2+1}{x^2}}}{\frac{x+3}{x}} = \lim_{x\to\infty} \frac{\sqrt{4+\frac{1}{x^2}}}{1+\frac{3}{x}} = \frac{\sqrt{4}}{1} = 2$$

$$\lim_{x\to-\infty} [2\ln(x+1) - \ln(x^2)] = \lim_{x\to-\infty} \ln\left(\frac{(x+1)^2}{x^2}\right) = \ln(1) = 0$$

For the final limit, we note that

$$\lim_{x\to-\infty} \frac{2}{\sqrt{x}} = 0$$

so then

$$\lim_{x\to\infty} \left[3\cos(x) - \frac{2}{\sqrt{x}}\right] = \lim_{x\to\infty} 3\cos(x)$$

and hence this limit does not exist since $\cos(x)$ oscillates between 1 and -1.

Question 2

Consider

$$f(x) = \frac{1}{x^2}$$

We calculate the derivative as follows.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-2hx-h^2}{x^2(x+h)^2}}{h} = \lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} = \frac{-2x}{x^4} = -\frac{2}{x^3}$$

Therefore,

$$f'(x) = -\frac{2}{x^3}$$

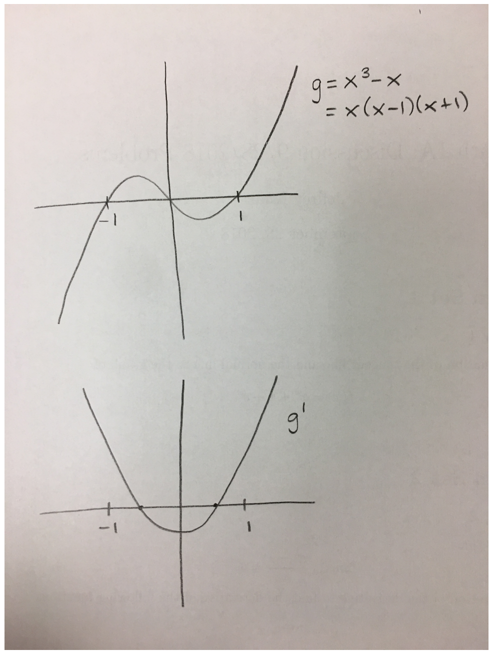

Figure 1: Graphs of g and g' for Question 3

We want to find the tangent line to the graph of $f(x)$ at $\left(\frac{1}{2}, 4\right)$. We need to find the slope of the tangent line, which is $f'(1/2)$. We have that

$$f'\left(\frac{1}{2}\right) = -\frac{2}{(1/2)^3} = -16$$

So the equation of the tangent line (using point-slope form) is

$$y - 4 = -16\left(x - \frac{1}{2}\right)$$

Question 3

The zeros of the polynomial $g(x) = x^3 - x = x(x-1)(x+1)$ are $x = -1, 0, 1$.

The derivative of $g(x)$ is

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - (x+h) - x^3 + x}{h}$$
$$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 - h}{h} = \lim_{h\to 0} 3x^2 + 3xh + h^2 - 1 = 3x^2 - 1$$

The graphs of g and g' are shown above.