Math 1A: Discussion 9/14/2018 Problems

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Problem Set 1

Question 1: Concept Check

Here is the definition of a limit again.

$$\lim_{x\to a} f(x) = L$$

is equivalent to:

$$\text{For every } \epsilon > 0, \text{ there exists } \delta > 0 \text{ such that } |f(x) - L| < \epsilon \text{ whenever } 0 < |x-a| < \delta$$

- The number ϵ represents the (horizontal/vertical) distance from _________.
- The number δ represents the (horizontal/vertical) distance from _________.
- The definition says that if we choose a (ϵ/δ), then we can find a (ϵ/δ) that satisfies the above definition.
- Why are we allowing every $\epsilon > 0$ in the definition?
- Why do we have $0 < |x - a| < \delta$ instead of $|x - a| < \delta$?
- Summarize the definition above in plain English. (No math expressions or symbols!)

Problem Set 2

Question 2

Use the ϵ-δ definition of the limit to show that

$$\lim_{x\to 1} (2x+1) = 3$$

Question 3

- Use the ϵ-δ definition of the limit to show that

$$\lim_{x\to 0} |x| = 0$$

- Now consider the function defined by

$$f(x) = x \text{ if } x > 0$$

$$f(x) = -x \text{ if } x < 0$$

$$f(x) = 1 \text{ if } x = 0$$

  Use the previous part to show that f is not continuous at $x = 0$. (This should be short!)

Problem Set 3

Question 4 (*)

- Use the Squeeze Theorem to show that

$$\lim_{x\to 0} \left[x \sin\left(\frac{1}{x^2}\right)\right] = 0$$

- Now use the ϵ-δ definition of a limit to show that

$$\lim_{x\to 0} \left[x \sin\left(\frac{1}{x^2}\right)\right] = 0$$

  (Hint: $-1 \leq \sin\left(\frac{1}{x^2}\right) \leq 1$ when $x \neq 0$. Why?)