Math 1A: Discussion 9/12/2018 Solutions

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September 12, 2018

Problem Set 1

Question 1

We have that $(3-2x)(3+2x)$ is a difference of squares:

$$(3-2x)(3+2x) = (3)^2 - (2x)^2 = 9 - 4x^2$$

We factor the quadratics as follows:

$$x^2 - 9 = (x+3)(x-3)$$

$$x^2 - 5x + 6 = (x-2)(x-3)$$

We have that the reference angle here is $\pi/3$ (the acute angle made with the x-axis). We have that $\pi/3$ is 60 degrees, so that

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

We note that $4\pi/3$ is in the third quadrant. So only tangent is positive here, and sine and cosine are negative. So

$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$$

$$\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$$

To simplify the sum of these two fractions, we place both fractions over a common denominator.

$$\frac{2}{1-x} + \frac{1}{2-x} = \frac{4-2x+1-x}{(1-x)(2-x)} = \frac{5-3x}{(1-x)(2-x)}$$

We have that

$$4^{1/2} = 2$$

so we have that

$$\frac{1}{8} = 2^{-3} = (4^{1/2})^{-3} = 4^{-3/2}$$

Therefore,

$$\log_4\left(\frac{1}{8}\right) = -\frac{3}{2}$$

Problem Set 2

Question 2

For the first limit, we plug in $x = 0$, and we get

$$\frac{\cos(0)}{1+2\sin(0)} = \frac{1}{1+0} = 1$$

Since all functions here are continuous, we get that the limit is just the value of the function at the point:

$$\lim_{x\to 0}\frac{\cos(2x)}{1+2\sin(x)} = 1$$

We plug in $h = 0$ for the next limit, and get $0/0$. So we have to do more work. Since there is a square root in the numerator, we should try to rationalize the numerator.

$$\lim_{h\to 0}\frac{\sqrt{1+h^2}-1}{h} = \lim_{h\to 0}\frac{\sqrt{1+h^2}-1}{h}\left(\frac{\sqrt{1+h^2}+1}{\sqrt{1+h^2}+1}\right)$$
$$= \lim_{h\to 0}\frac{(1+h^2)-1}{h(\sqrt{1+h^2}+1)} = \lim_{h\to 0}\frac{h^2}{h(\sqrt{1+h^2}+1)} = \lim_{h\to 0}\frac{h}{\sqrt{1+h^2}+1} = 0$$

We plug in $x = 2$ and we get $0/0$. So we have to do some work. We notice that we can simplify the numerator by using a common denominator.

$$\lim_{x\to 2}\frac{\frac{1}{(x-1)^2}-1}{x-2} = \lim_{x\to 2}\frac{1-x^2+2x-1}{(x-1)^2}\cdot\frac{1}{x-2} = \lim_{x\to 2}\frac{-x^2+2x}{(x-1)^2}\cdot\frac{1}{x-2}$$
$$= \lim_{x\to 2}\frac{-x(x-2)}{(x-1)^2}\cdot\frac{1}{x-2} = \lim_{x\to 2}\frac{-x}{(x-1)^2} = -2$$

For the next limit, we plug in $x = \pi$ and get $\pi^2/0$. So we have some sort of limit at infinity or negative infinity. We note that as x goes to π from the right, x^2 is basically around π^2. We next analyze the denominator. As x goes to π from the right, $\sin(x)$ is negative and getting smaller. So we have π^2 over something negative and getting smaller, so the result is negative and getting bigger. So therefore,

$$\lim_{x\to\pi^+}\frac{x^2}{\sin(x)} = -\infty$$

For the next limit, we plug in $x = 3$ and we get $0/0$. So we have to do more work. We note that we can factor so we can try that.

$$\lim_{x\to 3}\frac{x^2-6x+9}{x^2-9} = \lim_{x\to 3}\frac{(x-3)^2}{(x+3)(x-3)} = \lim_{x\to 3}\frac{x-3}{x+3} = 0$$

Now we consider the next limit

$$\lim_{x\to 0}\ e^{-x}\sin\left(x+\frac{5\pi}{6}\right)$$

The first step is to plug in. We get that

$$e^{0}\sin\left(\frac{5\pi}{6}\right)=\frac{1}{2}$$

So since all of our functions are well-behaved (in the sense that they are continuous), we have that the limit is

$$\lim_{x\to 0}\ e^{-x}\sin\left(x+\frac{5\pi}{6}\right)=\frac{1}{2}$$

Next, we consider the limit

$$\lim_{x\to 0^{+}}e^{x}\ln(x)$$

We plug in and have a problem because $\ln(x)$ is undefined at $x=0$. So we think about what happens as x approaches 0 from the right. We have that as x approaches 0 from the right, e^x is basically around 1. As x approaches zero from the right, $\ln(x)$ is getting negative and bigger and bigger. So multiplying a number that is around 1 by a number that is negative and getting bigger, we have that the the product is negative and getting bigger as x approaches 0 from the right. So

$$\lim_{x\to 0^{+}}e^{x}\ln(x)=-\infty$$

For the next limit, we plug in $x=\pi/4$. We get $(\pi^2/16)/0$, which suggests that we have a limit that is equal to infinity or negative infinity. As x approaches $\pi/4$ from the left, we have that x^2 is around $\pi^2/16$, so the numerator is basically just a positive constant. Next, we note that $\tan(x)$ approaches 1 and is less than 1 as x goes to $\pi/4$ from the left. So then $1-\tan(x)$ is positive and getting smaller and smaller as x goes to $\pi/4$ from the left. So as x goes to $\pi/4$ from the left, we have that $\frac{x^2}{1-\tan(x)}$ is basically a positive constant divided by a positive number that is getting smaller and smaller, which gives us a positive number that is getting bigger and bigger. So we have that

$$\lim_{x\to\frac{\pi}{4}^{-}}\frac{x^2}{1-\tan(x)}=\infty$$

The first step is to plug in $x=0$. We get

$$\frac{e^{e^{0}-1}}{1+e^{0}}=\frac{e^{0}}{1+e^{0}}=\frac{1}{2}$$

Because the functions involved are all well-behaved (in the sense that they are continuous), we have that

$$\lim_{x\to 0}\frac{e^{(e^{x}-1)}}{1+e^{x}}=\frac{1}{2}$$

For the next limit, we plug in $h=0$ and we get $0/0$. We note that there are square roots here so we try to rationalize the numerator.

$$\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2-h}}{h}=\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2-h}}{h}\left(\frac{\sqrt{2+h}+\sqrt{2-h}}{\sqrt{2+h}+\sqrt{2-h}}\right)$$

$$=\lim_{h\to 0}\frac{(2+h)-(2-h)}{h(\sqrt{2+h}+\sqrt{2-h})}=\lim_{h\to 0}\frac{2h}{h(\sqrt{2+h}+\sqrt{2-h})}=\lim_{h\to 0}\frac{2}{\sqrt{2+h}+\sqrt{2-h}}$$

$$=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$$

For the last limit, we plug in $h = 0$ and get $0/0$. We note that we can simplify the expression in the numerator by putting everything over a common denominator.

$$\lim_{h\to 0} \frac{\frac{1}{1+h} + \frac{1}{1-h} - 2}{h^2} = \lim_{h\to 0} \frac{\frac{1-h+1+h-(2-2h^2)}{1-h^2}}{h^2} = \lim_{h\to 0} \frac{2h^2}{1-h^2} \cdot \frac{1}{h^2} = \lim_{h\to 0} \frac{2}{1-h^2} = 2$$