Math 1A: Discussion 9/10/2018 Solutions

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Problem Set 1

Question 1

We show a graph of f below.

To calculate these limits, we first calculate the left and right hand limits. We have that

$$\lim_{t\to 0^-} f(t) = \lim_{t\to 0^-} (t-1) = -1$$

$$\lim_{t\to 0^+} f(t) = \lim_{t\to 0^+} t^2 = 0$$

So since the left and right hand limits are different, we see that

$$\lim_{t\to 0} f(t)$$

does not exist.

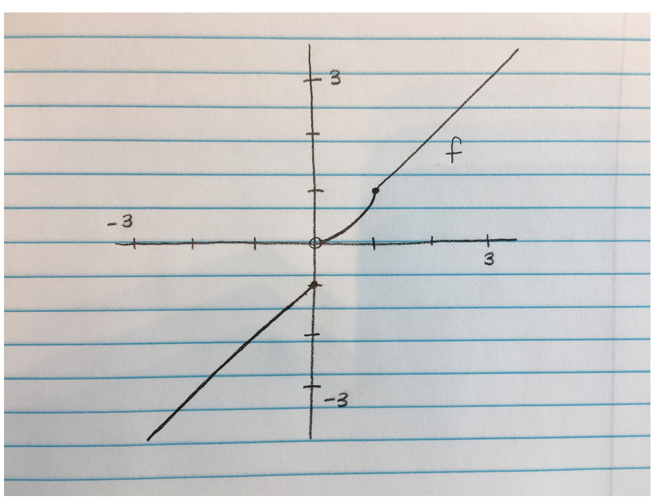
For the next limit, we have that

$$\lim_{t\to 1^-} f(t) = \lim_{t\to 1^-} t^2 = 1$$

$$\lim_{t\to 1^+} f(t) = \lim_{t\to 1^+} t = 1$$

So since both the left and right hand limits exist and are equal, we have that

$$\lim_{t\to 1} f(t) = 1$$



Question 2

For the first limit, the first step is to plug in.

$$0^2 - 3(0) + \sqrt{0+1} = 1$$

This answer makes sense, so since our function is well-behaved (continuous), we just have that

$$\lim_{x\to 0} \left(x^2 - 3x + \sqrt{x+1}\right) = 1$$

For the next limit, the first step is to plug in. We get 0/0, so we have to do some more work. We observe that we can factor the numerator.

$$\lim_{x\to 2} \frac{x^2 - 3x + 2}{x-2} = \lim_{x\to 2} \frac{(x-1)(x-2)}{x-2} = \lim_{x\to 2}(x-1) = 1$$

where we can cancel out the $x-2$ from the numerator and denominator because we do not care about what happens when $x = 2$ when evaluating the limit.

For the next limit, we plug in and get 2/0. So we see that we have some sort of vertical asymptote/limit at infinity. As we approach 2 from the left hand side, we note that $x-2$ is negative and getting smaller, while the numerator is around 2. So 2 over a small negative number is large and negative, so we have that

$$\lim_{x\to 2^-} \frac{x}{x-2} = -\infty$$

Now, when we approach 2 from the right hand side, $x-2$ is positive and getting smaller while the numerator is staying around 2. Since 2 divided by a small positive number is large and positive, we have that

$$\lim_{x\to 2^+} \frac{x}{x-2} = \infty$$

Problem Set 2

Question 3

Let us compute

$$\lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$$

The first step is to plug in $h = 0$. We get 0/0 so we need to do more work. We observe that we can simplify the numerator. Doing this reveals that there is an h/h that we can cancel out.

$$\lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{h} = \lim_{h\to 0} \frac{-h}{2(2+h)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-1}{2(2+h)} = -\frac{1}{4}$$

For the next limit, we plug in and immediately notice that $\tan\left(\frac{\pi}{2}\right)$ is undefined. So we have some sort of asymptote. As x approaches $\pi/2$ from the left, we have that $\tan(x)$ is positive and getting larger and larger, going to infinity. Meanwhile, the denominator is around $\pi/2$. So we have something positive and getting larger and larger, divided by something that is around a constant. So we conclude that

$$\lim_{x\to \frac{\pi}{2}^-} \frac{\tan(x)}{x} = \infty$$

For the next limit, we plug in and get $1/0$, so we have a vertical asymptote/limit at infinity. We note that as we approach 0 from the right hand side, e^x is approaching 1 and is actually greater than 1. So as we approach 0 from the right hand side, we have that $1 - e^x$ is getting closer and closer to 0 but is negative. So 1 divided by a negative number that is getting smaller and smaller (closer and closer to 0) gives a negative number that is getting larger and larger. So

$$\lim_{x\to 0^+} \frac{1}{1-e^x} = -\infty$$

For the next limit, we plug in $h = 0$ and we get $0/0$, so we have to do more work. We rationalize the numerator and see that doing this makes an h/h appear that we can cancel.

$$\begin{aligned}\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} &= \lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}\left(\frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}\right) \\ &= \lim_{h\to 0} \frac{(1+h)-1}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{h}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{2}\end{aligned}$$

For the next limit, we plug in $x = 1$ and get $0/0$ so more work needs to be done. Even though it doesn't look like it at first, the denominator can actually be written as a difference of squares. This will let us cancel out the $1 - \sqrt{x}$ from the numerator.

$$\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} = \lim_{x\to 1} \frac{1-\sqrt{x}}{(1+\sqrt{x})(1-\sqrt{x})} = \lim_{x\to 1} \frac{1}{1+\sqrt{x}} = \frac{1}{2}$$

Problem Set 3

Question 4 (*)

We first plug in $h = 0$ and see that we get $0/0$. So we have to do more work. Here, we will use the difference of cubes formula:

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Doing so to "rationalize" the numerator will make an h/h appear which we can cancel.

$$\begin{aligned}\lim_{h\to 0} \frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h} &= \lim_{h\to 0} \frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}\left(\frac{(\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\,\sqrt[3]{x}+(\sqrt[3]{x})^2}{(\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\,\sqrt[3]{x}+(\sqrt[3]{x})^2}\right) \\ = \lim_{h\to 0} \frac{(x+h)-(x)}{h((\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\,\sqrt[3]{x}+(\sqrt[3]{x})^2)} &= \lim_{h\to 0} \frac{h}{h((\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\,\sqrt[3]{x}+(\sqrt[3]{x})^2)} \\ = \lim_{h\to 0} \frac{1}{(\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\,\sqrt[3]{x}+(\sqrt[3]{x})^2} &= \frac{1}{(\sqrt[3]{x})^2+\sqrt[3]{x}\,\sqrt[3]{x}+(\sqrt[3]{x})^2} = \frac{1}{3x^{2/3}}\end{aligned}$$