Math 1A: Discussion 10/5/2018 Problems

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Problem Set 1

Question 1

Find the derivatives of the following functions.

$$f(x) = \frac{\sin(2x)}{1+\cos(2x)}$$

$$f(x) = e^{-3x} + \tan(x)$$

$$f(x) = \sqrt{e^x + \sin(x)} + xe^{2x}$$

Question 2

Calculate each of the following derivatives in the multiple ways described and check that all methods give the same answer.

- $f(x) = \sin^2(x)$, using (1) the chain rule, and (2) then using the product rule (by writing $f(x) = \sin(x) \cdot \sin(x)$).
- $f(x) = \sin^2(x) + \cos^2(x)$, using (1) the chain rule twice, and (2) using the fact that $\sin^2(x) + \cos^2(x) = 1$.
- $f(x) = \frac{1+x}{x^2}$, using (1) the quotient rule, and (2) simplifying and using the power rule.
- $f(x) = e^{2x}$, using (1) the chain rule with $f(x) = e^{(2x)}$, and (2) the chain rule with $f(x) = (e^x)^2$.

Problem Set 2

Question 3

- Find the equation of the tangent line to

  $$g(x) = e^x \sec(x^2 + x)$$

  at $x = 0$.
- Find the derivative of the function

  $$h(x) = e^{(e^x)} \sin(\sin(x))$$

  Find the equation of the tangent line to h at $x = 0$.

Question 4

Find the 20th derivative of

$$f(x) = \frac{1}{1-x}$$

Then, find the 20th derivative of

$$f(x) = \frac{1}{1-5x}$$

Problem Set 3

Question 5 (*)

For each of the following equations, find a function f that satisfies the equation.

$$f' = 3f$$

$$f''' = -8f$$

$$f'' = -\frac{1}{4}f$$

$$\frac{f'}{f} = \frac{2}{x}$$

$$\frac{f'}{f} = 2x$$

(Hint: None of the functions here are complicated. Use the chain rule and the power rule.)

Question 6 (**)

Let f be a function that has an inverse f^{-1}. Then,

$$f(f^{-1}(x)) = x$$

- By taking the derivative of both sides of the equation $f(f^{-1}(x)) = x$ with respect to x, find a formula for

  $$\frac{d}{dx}(f^{-1}(x))$$

  (Hint: When taking the derivative of the equation with respect to x, you will have to use the chain rule carefully to take the derivative of the left hand side of the equation with respect to x).

- By using your formula from the previous part, find

  $$\frac{d}{dx}(\arcsin(x))$$

  $$\frac{d}{dx}(\arctan(x))$$