

# Math 1A: L'Hopital's Rule and Exponential Growth/Decay

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October 31, 2018

## L'Hopital's Rule

### Indeterminate Quotients

$$\lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{x^2 - 4}$$

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{4\sqrt{x}}$$

$$\lim_{x \rightarrow 1} \frac{x^2 + 1}{x - 1}$$

### Indeterminate Products

$$\lim_{x \rightarrow 1} (x^2 - 1)\ln(x - 1)$$

$$\lim_{x \rightarrow 0^+} \ln(x)\arctan(x)$$

### Indeterminate Differences

$$\lim_{x \rightarrow \infty} (\ln(x) - x^2)$$

$$\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{\arctan(x)} \right)$$

### Indeterminate Powers

$$\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{\sqrt{x}} \right)^{x^2}$$

$$\lim_{x \rightarrow 0^+} \left( \frac{1}{x} \right)^{x^2+x}$$

$$\lim_{x \rightarrow 0^+} x^{(\sqrt[3]{x})}$$

## Exponential Growth and Decay

### Exponential Growth

The population of Pallet Town grows exponentially. Suppose that Pallet Town has 100 residents in the year 2000 (including the one and only Ash Ketchum) and 150 residents in the year 2010. Let  $t$  be the number of years after 2000. Write a formula for the population of Pallet town  $P(t)$  as a function of  $t$  assuming that the population continues to grow exponentially.

### Exponential Decay

Fifty years pass and the population of Pallet Town is now 500 people. However, in the year 2050, a new set of high-rise apartments in Viridian City with luxury apartments for Pokemon trainers and their Pokemon causes people to move out of Pallet Town. Starting in the year 2050, the population of Pallet Town decays exponentially with a half life of 4 years as people move out. Write a formula for the population of Pallet town  $P(t)$  as a function of  $t$  where  $t$  now is the number of years after 2050.

### Newton's Law of Cooling

Ash's Charizard uses flamethrower to heat up a gold nugget Ash has found during his adventures. The gold nugget heats up to a temperature of 200 degrees Celcius (they use SI units in the Kanto region) and the ambient temperature is 27 degrees Celcius. After twenty minutes, the gold nugget has cooled to a temperature of 123 degrees Celcius. Given that the laws of physics still hold in the world of Pokemon, what is the temperature of the gold nugget after one hour?

### Compounded Interest

Ash, even though he is young, is a financially savvy Pokemon trainer. He decides to invest his money in the bank. He wants to invest 10000 Pokedollars, and he can choose between the following three options.

- OPTION A: Invest at a rate of 3 percent annually.
- OPTION B: Invest at a rate of 2.5 percent quarterly.
- OPTION C: Invest at a rate of 2 percent continuously.

Ash, being very precocious, also has a surprisingly good grasp of calculus (when he's not beating other Pokemon trainers in Pokemon battles, he's hard at work with paper, pencil, and his own copy of Stewart's *Calculus!*). He calculates which option yields the most money - and chooses it. Which option did he choose, and how much money does he gain after a year of investing?