

Math 1A: Discussion 10/3/2018 Problems

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October 3, 2018

$$(f(x)g(x))' = f(x)g'(x) + f'(x)g(x)$$

First times the derivative of the second plus the second times the derivative of the first.

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

Bottom times the derivative of the top minus the top times the derivative of the bottom
all over the bottom squared

$$\begin{aligned}\frac{d}{dx}(\sin(x)) &= \cos(x) \\ \frac{d}{dx}(\cos(x)) &= -\sin(x)\end{aligned}$$

Problem Set 1

Question 1

A student is taking his Math 1A Midterm 2, and comes across a question where he has to take a derivative. “I got this!” he says at first, but to his horror, he sees that as part of this derivative, he has to take the derivative of $y = \sec(x)$ and $y = \tan(x)$.

He thinks to himself, “Not more trigonometry!” and sighs... He vaguely remembers something about the derivative of $y = \tan(x)$ being... well, actually he realizes he doesn't remember at all. Oops.

But he suddenly has a burst of inspiration. “Maybe I can use the quotient rule...” he thinks. How can he calculate the derivatives of $y = \sec(x)$ and $y = \tan(x)$ on the spot?

Question 2

Calculate the derivative of

$$f(x) = \frac{\sin(x)}{x^2}$$

by first using the quotient rule, and then by using the product rule, by writing $f(x)$ as $\sin(x) \cdot \frac{1}{x^2}$. Check that your answers both of these ways give the same answer.

Problem Set 2

Question 3

Consider the function

$$f(x) = \frac{2x^2}{x^2 - 4}$$

- What are the vertical asymptotes of f ?
- What is the horizontal asymptote of f ?
- What are the zeros of f ?
- Calculate f' . What are the zeros, vertical asymptotes, and horizontal asymptotes of f' ?

Question 4

- What is the 1003rd derivative of $f(x) = \sin(x) - \cos(x)$?
- What is the 1003rd derivative of $f(x) = xe^x$?

Question 5

Find the derivatives of the following functions.

$$f(x) = \frac{\sin(x)}{xe^x}$$

$$g(x) = \sqrt{x} \cdot e^x \cos(x)$$

$$h(x) = \frac{\sqrt[3]{x}}{(x^2 + 3x - 1)e^x}$$

In addition, find the equation of the tangent line to h at $x = 0$.

Problem Set 3

Question 6 (*)

Consider the functions

$$f(x) = \sqrt{x} \sin(x)$$

$$g(x) = \frac{1}{\sqrt{x}} \sin(x)$$

Compute f' and g' , and find

$$\lim_{x \rightarrow \infty} f(x)$$

$$\lim_{x \rightarrow \infty} f'(x)$$

$$\lim_{x \rightarrow \infty} g(x)$$

$$\lim_{x \rightarrow \infty} g'(x)$$