Math 1A: Discussion 10/19/2018

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Question 1

Analyze the functions and curve sketch. A complete analysis includes finding

- The domain of the function
- The zeros of the function, and the y-intercept
- Vertical asymptotes, horizontal asymptotes, and slant asymptotes, if applicable (only for rational functions)
- Where the function is increasing or decreasing
- The location of all local maxima and minima
- Where the function is concave up and concave down
- The location of all inflection points

$$f(x) = \arctan(x^2)$$

(Midterm 2, 2001)

$$f(x) = x^3 - x^2$$

$$f(x) = \frac{x^3}{x^2 - 1}$$

(Midterm 2, 2005)

$$f(x) = x^2 e^x$$

(Midterm 2, 2005)

$$f(x) = x^{1/x} \text{ for } x > 0$$

Question 2

The following question is a past exam question from the UC Berkeley Ph.D Preliminary Examination, Spring 1984, taken by incoming Ph.D students. But you actually have the mathematical tools needed to solve this question already!

$$\text{Which number is larger, } \pi^3 \text{ or } 3^\pi?$$

If you want to try this question without any hints, you are welcome to try it! On the next page, there are steps that will walk you through how to do this question if you get stuck.

- Curve sketch the function
$$f(x) = \frac{\ln(x)}{x}$$
answering all the questions given in Question 1. This does not seem related to the question given above but it will be.

- Use your curve sketch to figure out which quantity is larger
$$\frac{\ln(3)}{3} \text{ or } \frac{\ln(\pi)}{\pi}$$

- Use the previous part to figure out which quantity is larger
$$\pi \ln(3) \text{ or } 3 \ln(\pi)$$

- Use the fact that e^x is strictly increasing to figure out whether π^3 or 3^π is larger.

Question 3

- Show that the function $\cos(x) + 2x$ has exactly one zero.
- Show that the function $y = x^3 + 3x^2 + 3x + 10$ has exactly one zero.
- (Midterm 2, 1995) Let c be a real number. Use Rolle's Theorem to show that the equation $x^5 - 6x + c = 0$ has at most one solution in the interval $[-1, 1]$.