Discussion 10/17/18

1) $\cdot\ x^2 - xy + y^2 = 1$

$$\frac{d}{dx}(x^2 - xy + y^2) = \frac{d}{dx}(1)$$

$$2x - \underbrace{\left(x\frac{d}{dx}(y) + \frac{d}{dx}(x)\cdot y\right)}_{\text{Product Rule}} + 2y\frac{dy}{dx} = 0$$

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y - x) = y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{2x - y}{x - 2y}}$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{2(1) - 1}{1 - 2(1)} = \boxed{-1}$$

$\cdot\ x^3y + x^2y^2 = 1 - y^4$

$$\frac{d}{dx}(x^3y + x^2y^2) = \frac{d}{dx}(1 - y^4)$$

$$x^3\frac{d}{dx}(y) + y\frac{d}{dx}(x^3) + x^2\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x^2)$$
$$= 0 - 4y^3\frac{dy}{dx}$$

$$x^3\frac{dy}{dx} + 3x^2y + 2x^2y\frac{dy}{dx} + 2xy^2 = -4y^3\frac{dy}{dx}$$

$$(x^3 + 2x^2y + 4y^3)\frac{dy}{dx} = -3x^2y - 2xy^2$$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2y - 2xy^2}{x^3 + 2x^2y + 4y^3}}$$

$$\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{-3(1)^2(-1) - 2(1)(-1)^2}{(1)^3 + 2(1)^2(-1) + 4(-1)^3} = \frac{3-2}{1-2-4} = \boxed{-\frac{1}{5}}$$

- $\ln(x)\ln(y)=1$

$$\frac{d}{dx}\left(\ln(x)\ln(y)\right)=\frac{d}{dx}(1)$$

$$\ln x\frac{d}{dx}(\ln y)+\ln y\frac{d}{dx}(\ln x)=0$$

(Product Rule)

$$(\ln x)\cdot\frac{1}{y}\frac{dy}{dx}+(\ln y)\cdot\frac{1}{x}=0$$

$$(\ln x)\cdot\frac{1}{y}\frac{dy}{dx}=-\frac{1}{x}\ln y$$

$$\boxed{\frac{dy}{dx}=-\frac{y\ln y}{x\ln x}}$$

$$\left.\frac{dy}{dx}\right|_{(e,e)}=-\frac{e\ln e}{e\ln e}=\boxed{-1}$$

- $\sin x\arctan y+e^{xy}=1$

$$\frac{d}{dx}\left(\sin x\arctan y+e^{xy}\right)=\frac{d}{dx}(1)$$

$$\sin x\frac{1}{1+y^2}\frac{dy}{dx}+\cos x\arctan y+\underbrace{e^{xy}\frac{d}{dx}(xy)}_{\text{Chain Rule}}=0$$

$$\frac{\sin x}{1+y^2}\frac{dy}{dx}+\cos x\arctan y+xe^{xy}\frac{dy}{dx}+ye^{xy}=0$$

$$\frac{dy}{dx}\left(\frac{\sin x}{1+y^2}+xe^{xy}\right)=-\cos x\arctan y-ye^{xy}$$

$$\boxed{\frac{dy}{dx}=\frac{-\cos x\arctan y-ye^{xy}}{\frac{\sin x}{1+y^2}+xe^{xy}}}$$

$\cdot \sqrt{x}\, e^{\sin y} + x^2 e^{2y} + \tan y = 0$

$$\frac{d}{dx}\left(\sqrt{x}\, e^{\sin y} + x^2 e^{2y} + \tan y\right) = 0$$

$$\sqrt{x} \cos y\, e^{\sin y} \frac{dy}{dx} + \frac{1}{2\sqrt{x}} e^{\sin y} + 2x^2 e^{2y} \frac{dy}{dx} + 2xe^{2y}$$
$$+ \sec^2 y \cdot \frac{dy}{dx} = 0.$$

$$\left(\sqrt{x} \cos y\, e^{\sin y} + 2x^2 e^{2y} + \sec^2 y\right) \frac{dy}{dx}$$
$$= -\frac{1}{2\sqrt{x}} e^{\sin y} - 2xe^{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-\frac{1}{2\sqrt{x}} e^{\sin y} - 2xe^{2y}}{\sqrt{x} \cos y\, e^{\sin y} + 2x^2 e^{2y} + \sec^2 y}}$$

2) $\cdot f(x) = x^{\sqrt{x}} \longrightarrow \ln(f(x)) = \ln(x^{\sqrt{x}})$

$$\ln(f(x)) = \sqrt{x} \ln(x)$$
$$\ln(y) = \sqrt{x} \ln(x)$$
$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\sqrt{x} \ln x)$$
$$\frac{1}{y} \frac{dy}{dx} = \frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}}$$
$$\frac{dy}{dx} = y\left(\frac{1}{\sqrt{x}}\left(1 + \frac{1}{2}\ln x\right)\right)$$

Substitute $y = x^{\sqrt{x}}$.

$$\boxed{\frac{dy}{dx} = x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}}\left(1 + \frac{\ln x}{2}\right)\right)}$$

$\cdot\ f(x) = (x^2+1)^{\cos x}$

$$y = (x^2+1)^{\cos x}$$
$$\ln(y) = \ln((x^2+1)^{\cos x})$$

$$\ln(y) = \cos x \ln(x^2+1)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(\cos x \ln(x^2+1))$$
$$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \frac{2x}{x^2+1} - \sin x \cdot \ln(x^2+1)$$
$$\frac{dy}{dx} = y\left(\frac{2x\cos x}{x^2+1} - \sin x \ln(x^2+1)\right)$$

$$\boxed{\frac{dy}{dx} = (x^2+1)^{\cos x}\left(\frac{2x\cos x}{x^2+1} - \sin x \ln(x^2+1)\right)}$$

$\cdot\ f(x) = (2+\arctan x)^{\arctan x}$

$$y = (2+\arctan x)^{\arctan x}$$
$$\ln(y) = \ln((2+\arctan x)^{\arctan x})$$
$$\ln(y) = \arctan x \ln(2+\arctan x)$$
$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}((\arctan x \ln(2+\arctan x))$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{\arctan x}{2+\arctan x} \cdot \frac{1}{1+x^2} + \frac{\ln(2+\arctan x)}{1+x^2}$$
$$\frac{dy}{dx} = y\left(\frac{\arctan x}{2+\arctan x} + \ln(2+\arctan x)\right)\frac{1}{1+x^2}$$
$$\boxed{\frac{dy}{dx} = (2+\arctan x)^{\arctan x}\left(\frac{\arctan x}{2+\arctan x} + \ln(2+\arctan x)\right)\frac{1}{1+x^2}}$$

$\cdot\ f(x) = (x^4+1)^{(\ln x + e^x)}$

$$y = (x^4+1)^{(\ln x + e^x)}$$
$$\ln(y) = \ln\left((x^4+1)^{(\ln x + e^x)}\right)$$
$$\ln(y) = (\ln x + e^x)\ln(x^4+1)$$
$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left((\ln x + e^x)\ln(x^4+1)\right)$$
$$\frac{1}{y}\frac{dy}{dx} = (\ln x + e^x)\frac{4x^3}{x^4+1} + \left(\frac{1}{x} + e^x\right)\ln(x^4+1)$$
$$\frac{dy}{dx} = y\left[(\ln x + e^x)\frac{4x^3}{x^4+1} + \left(\frac{1}{x} + e^x\right)\ln(x^4+1)\right]$$
$$\boxed{\frac{dy}{dx} = (x^4+1)^{\ln x + e^x}\left[(\ln x + e^x)\frac{4x^3}{x^4+1} + \left(\frac{1}{x} + e^x\right)\ln(x^4+1)\right]}$$

$\cdot\ f(x) = x^{(x^x)}$

$$y = x^{(x^x)}$$
$$\ln y = \ln\left(x^{(x^x)}\right)$$
$$\ln y = x^x \ln(x) \longrightarrow \text{ASIDE: What is } (x^x)'?$$
$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x^x \ln x)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x^x)\cdot \ln x + \frac{x^x}{x}$$
$$\frac{dy}{dx} = y\left[x^x(1+\ln x)\ln x + \frac{x^x}{x}\right]$$
$$\boxed{\frac{dy}{dx} = x^{(x^x)}\left[x^x(1+\ln x)\ln x + x^{x-1}\right]}$$

ASIDE:
$$y = x^x$$
$$\ln y = \ln(x^x)$$
$$\ln y = x \ln x$$
$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \ln x)$$
$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x$$
$$\frac{dy}{dx} = y(1+\ln x)$$
$$\frac{dy}{dx} = x^x(1+\ln x)$$
$$(x^x)' = x^x(1+\ln x)$$