# Math 1A: Discussion 10/1/2018 Problems

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## Problem Set 1

### Question 1

(Midterm 2, Fall 2005, Question 1)

For what values of $x$ does the graph of $f(x) = x - 2\sin(x)$ have a horizontal tangent?

## Problem Set 2

### Question 2

(Midterm 2, Fall 1995, Question 1)

Evaluate

$$\lim_{b \to 2} \frac{b^{691} - 2^{691}}{b - 2}$$

by using rules of differentiation, first expressing the limit as a derivative.

### Question 3

- (Midterm 2, Fall 2012, Question 4)

  Use a linear approximation estimate $\sqrt[4]{1.003}$.

- Approximate $e^{0.05}$.

### Question 4

We can take multiple derivatives in the following way. Let the second derivative of a function $f$ be the derivative of $f'$. More generally, let the $n$th derivative of $f$ be the result you get after taking the derivative of $f$ a total of $n$ times in a row. For example, to find the third derivative of $f(x) = \frac{1}{2}x^4$, we take three derivatives of $f$ in a row:

$$\frac{1}{2}x^4 \to 2x^3 \to 6x^2 \to 12x$$

So the third derivative of $f$, denoted by $f'''(x)$ or $f^{(3)}(x)$, is

$$f'''(x) = f^{(3)}(x) = 12x$$

- What is the tenth derivative of $y = e^{2x}$?
- What is the fiftieth derivative of $y = x^{50}$?
- What is the 101st derivative of $y = x^{100} + e^x$?

# Problem Set 3

## Question 5

In this question, we will examine why a function can fail to be differentiable. Consider the two functions

$$f(x) = |x|$$
$$g(x) = x^{2/3}$$

- Quickly sketch $f$ and $g$.
- Show (using the definition of the derivative) that $f(x)$ and $g(x)$ are both not differentiable at $x = 0$.
- Calculate $f'(x)$ and $g'(x)$, when they exist. Then, find the following limits

$$\lim_{x\to 0^+} f'(x)$$
$$\lim_{x\to 0^-} f'(x)$$
$$\lim_{x\to 0^+} g'(x)$$
$$\lim_{x\to 0^-} g'(x)$$

- We say that $f$ has a corner at $x = 0$, and $g$ has a cusp or vertical tangent at $x = 0$. Interpret your result from the previous part to explain why these terms make sense.
- Consider the functions $y = e^{-|x|}$, $y = \sqrt{|x|}$, and $y = e^{|x|} - \frac{1}{2}x^2$. State whether these functions are differentiable at $x = 0$. If a function fails to be differentiable at $x = 0$, state whether the function has a corner or a cusp/vertical tangent there.