

# Math 1A: Discussion 11/14/18

Jeffrey Kuan

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## Problem Set 1

### Question 1

Evaluate the following integrals.

- Hint for the fourth and fifth integrals: use trigonometric identities
- Hint for the last integral: partial fraction decomposition, write the integrand as

$$\frac{1}{x^2 - 1} = \frac{C_1}{x - 1} + \frac{C_2}{x + 1}$$

and use this to find an antiderivative.

$$\int_{-1}^1 \frac{2}{1+x^2} dx$$
$$\int_1^2 \left( x^2 + 1 + \frac{1}{x^2} \right) dx$$
$$\int_{-e^2}^{-1} \left( \frac{2}{x} + \frac{3}{x^2} \right) dx$$
$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^3(\theta) \cos(\theta) d\theta$$
$$\int_0^{\frac{\pi}{4}} \sec^2(x) e^{\tan(x)} dx$$
$$\int_{-1/2}^{1/2} \frac{1}{x^2 - 1} dx$$

## Problem Set 2

### Question 2

Calculate

$$\frac{d}{dx} \int_0^x \sin^3(y) dy$$

$$\frac{d}{dx} \int_x^2 \sin^3(y) dy$$

$$\frac{d}{dx} \int_{-1}^{x^2+\sqrt{x}} \sin^3(y) dy$$

$$\frac{d}{dx} \left( e^x \int_{x^2}^{x^3} \sin^3(y) dy \right)$$

### Question 3 (Midterm 2 Reprise)

Let  $f$  be the continuous function given by

$$f(x) = x \ln(|x|) \text{ for } x \neq 0$$

$$f(x) = a \text{ for } x = 0$$

where  $a$  is a constant that makes  $f$  continuous everywhere.

- What is the value of  $a$ ? (Hint: L'Hopital)
- Consider the function

$$F(x) = \int_0^x f(x) dx$$

Find all relative maxima and minima of  $F$ , all points of inflection, and use this information to graph the function  $F$ .

## Problem Set 3

### Question 4 (\*)

Let  $f$  be a continuous function on  $[a, b]$ . Then, note that

$$F(x) = \int_a^x f(x) dx$$

is a differentiable function on  $[a, b]$ . Apply the Mean Value theorem to  $F$  to deduce the **mean value theorem for integrals**. Then, illustrate the mean value theorem for integrals using the function  $f(x) = \sqrt{x}$  on the interval  $[0, 4]$ .