Math 1A: Discussion 8/27/2018 Solutions

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1 Problem Set 1

1.1 Question 1

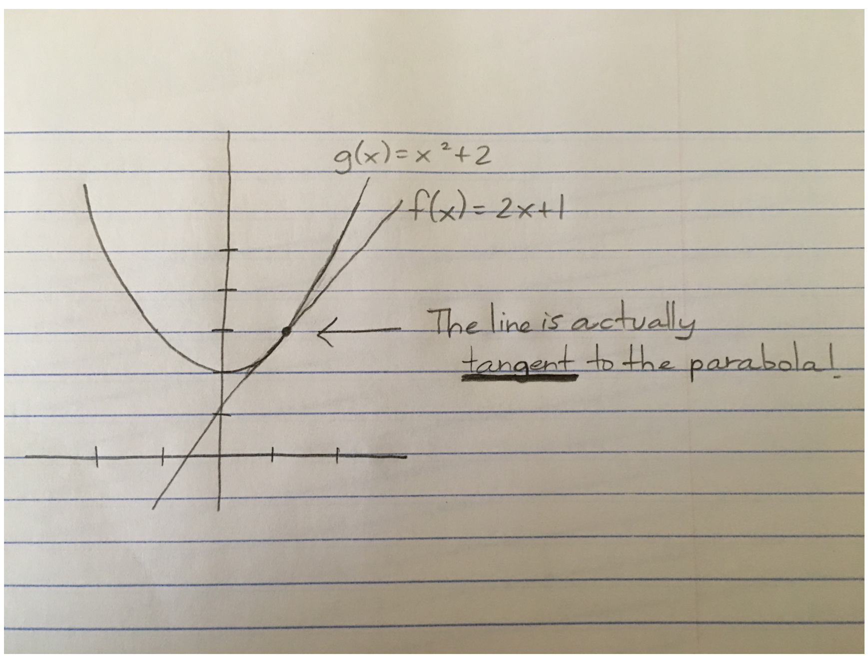
We have that

$$f(1) = 2(1) + 1 = 3$$

and

$$g(0) = 0^2 + 2 = 2$$

The graphs of f and g are shown below.



To find whether these functions are even or odd, consider $f(-x)$ and $g(-x)$.

$$f(-x) = 2(-x) + 1 = -2x + 1$$

So $f(-x)$ is not the same as $f(x)$ or $-f(x)$. So f is **neither even nor odd**.

We have that

$$g(-x) = (-x)^2 + 2 = x^2 + 2$$

So we see that $g(-x)$ is equal to $g(x)$, and not equal to $-g(x)$. So g is **even but not odd**.

A graph will show that f is increasing on $\mathbb{R}$. The function g is decreasing on $(-\infty, 0]$ and increasing on $[0, \infty)$.

To find where $f(x) = g(x)$, we set $2x + 1 = x^2 + 2$. Solving, we get

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

So $f(x) = g(x)$ when $x = 1$.

2 Problem Set 2

2.1 Question 2

To find the domain of f and g, we want to find all points where the denominator is not zero, as a fraction cannot have a denominator equal to 0. So the domain of f and g are both $(-\infty, 3) \cup (3, \infty)$.

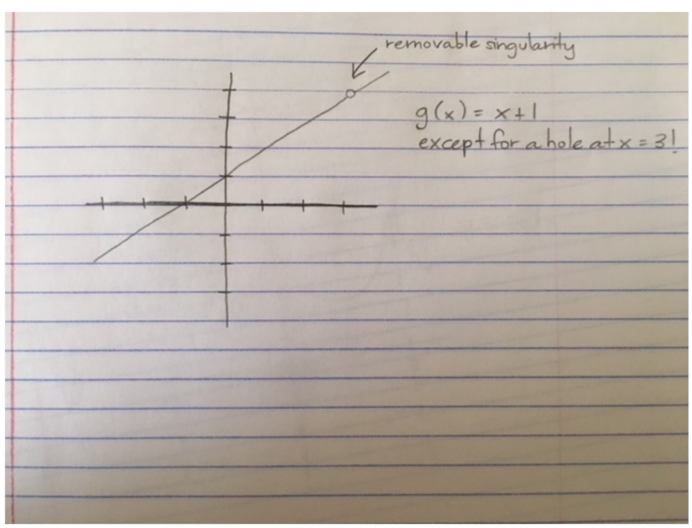
To find points where $f(x) = 0$, we set $x^2 + 2x + 1 = 0$, and see that this is the same as $(x + 1)^2 = 0$. So the only x that satisfies this is $x = -1$, and the denominator is indeed nonzero there. So $f(x) = 0$ at $x = -1$.

For $g(x) = 0$, we set $x^2 - 2x - 3 = 0$, and get $(x - 3)(x + 1) = 0$. Then, $x = -1, 3$. But note that at $x = 3$, the function $g(x)$ is undefined. So we have $g(x) = 0$ only at $x = -1$.

The equation for $g(x)$ can be simplified as

$$g(x) = \frac{(x + 1)(x - 3)}{x - 3}$$

So $g(x) = x + 1$ when $x \neq 3$ and is undefined for $x = 3$. (Important! We are canceling $x - 3$ here, but you can only cancel when the thing you are canceling is nonzero. So the formula $g(x) = x + 1$ only works if $x \neq 3$). So $g(x)$ is just a line with a hole at $x = 3$.



There is no value of x with $f(x) = 2$ because if we solve the equation, we get

$$\frac{x^2 + 2x + 1}{x - 3} = 2$$

$$x^2 + 2x + 1 = 2x - 6$$

$$x^2 = -7$$

But this has no solutions in $\mathbb{R}$. So there is no value of x that has $f(x) = 2$.

2.2 Question 3

Just substitute the values in and simplify. Remember that x and h are both variables, so just simplify as much as you can.

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-(x+h)^2 + 2(x+h) - (-x^2 + 2x)}{h} \\ &= \frac{-x^2 - 2hx - h^2 + 2x + 2h + x^2 - 2x}{h} = \frac{-2hx - h^2 + 2h}{h} = -2x + 2 - h\end{aligned}$$

2.3 Question 4

For the function $f(x) = \sqrt{x^2 - 1}$, to find the domain, we set

$$x^2 - 1 \geq 0$$

$$(x-1)(x+1) \geq 0$$

Trying values in $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$, we see that the solution is $x \leq -1$, $x \geq 1$. So the domain of f is

$$(-\infty, -1] \cup [1, \infty)$$

Meanwhile, the domain of $g(x) = \sqrt{x^2 + 1}$ is all real numbers. We want to solve the inequality

$$x^2 + 1 \geq 0$$

But we note that for all real numbers x, we have that $x^2 \geq 0$. So $x^2 + 1 \geq 0$ also, and thus, g is defined for all real numbers.

2.4 Question 5

The volume of a cylinder is $V = \pi r^2 h$. So we have that

$$V(h) = \pi \left(\frac{h}{4}\right)^2 h = \frac{\pi}{16} h^3$$

The surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$. So we have that

$$S(r) = 2\pi r^2 + 2\pi r(4r) = 10\pi r^2$$

3 Problem Set 3

3.1 (*) Question 6

We set the denominator equal to 0. We get that the function is undefined whenever $\sin(x) = 0$, so the domain is all real numbers, excluding multiples of π.

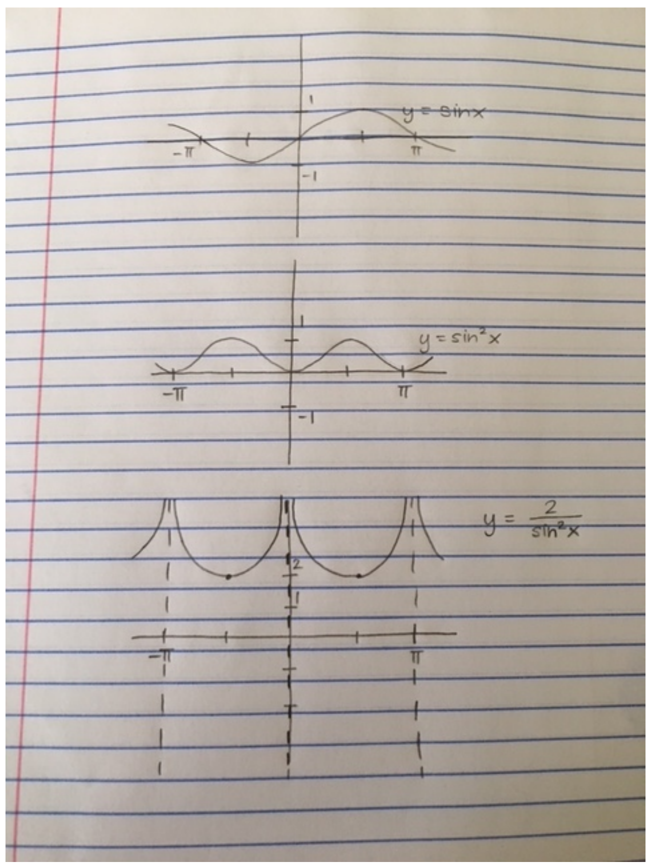
For the range, we note that the range of $\sin(x)$ is $[-1, 1]$, so the range of $\sin^2(x)$ is $[0, 1]$. Note that on the domain of h though, $\sin^2(x) \neq 0$, so on the domain of h, $\sin^2(x)$ takes on values in $(0, 1]$. So then, $h(x)$ takes on values in $[2, \infty)$. So the range of h is $[2, \infty)$.

We recall that $\sin(-x) = -\sin(x)$. So then

$$h(-x) = \frac{2}{(\sin(-x))^2} = \frac{2}{(-\sin(x))^2} = \frac{2}{\sin^2(x)}$$

So we see that $h(-x) = h(x)$, and $h(-x)$ is not the same as $-h(x)$. So h is **even but not odd.**

A graph of h can be obtained by successively graphing intermediate functions, as shown below.



3.2 (**) Question 7

We note that the only function on $\mathbb{R}$ that is both even and odd is the zero function. To see this, suppose that $h(x)$ is a function on $\mathbb{R}$ that is both even and odd. Then, for every $x \in \mathbb{R}$, we have that $h(-x) = h(x) = -h(x)$. So $h(x) = -h(x)$ for all real numbers, and hence $h(x) = 0$ for all real numbers. This shows that the only function on $\mathbb{R}$ that is both even and odd is the zero function.

We are given that $f(x)$ is even, so then we have that $f(x) - 1$ is also even. (This is because then $f(-x) - 1 = f(x) - 1$). So $f(x) - 1$ is a function on $\mathbb{R}$ that is both even and odd. So by our previous observation, $f(x) - 1$ is the zero function, so $f(x)$ is the constant function that is equal to 1 everywhere.