

Math 1A: Discussion 8/27/2018

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General Comments: I will organize discussion worksheets by separating problems into three sets of problems. The problems in Problem Set 1 are intended to be more basic, the problems in Problem Set 2 are of medium difficulty, and the problems in Problem Set 3 are intended to be challenge questions (with (*) denoting a difficult question and (**) denoting a very difficult question). If you are able to answer the questions in Problem Sets 1 and 2, that is sufficient (but of course, it is good to try the challenge questions too!).

1 Problem Set 1

1.1 Question 1

Consider the functions $f(x) = 2x + 1$ and $g(x) = x^2 + 2$, defined on \mathbb{R} .

- What is $f(1)$? What is $g(0)$?
- Sketch the graphs of f and g on the same coordinate plane.
- Is f even, odd, both even and odd, or neither? Is g even, odd, both even and odd, or neither?
- Where is f increasing/decreasing? Where is g increasing/decreasing?
- Where does $f(x) = g(x)$?

2 Problem Set 2

2.1 Question 2

Consider the following functions:

$$f(x) = \frac{x^2 + 2x + 1}{x - 3}$$
$$g(x) = \frac{x^2 - 2x - 3}{x - 3}$$

- What is the domain of f ? What is the domain of g ?
- For which values of x do we have $f(x) = 0$ and $g(x) = 0$?
- Sketch the graph of $g(x)$.
- Is there a value of x such that $f(x) = 2$? Justify your answer.

2.2 Question 3

Evaluate the difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

for the function $f(x) = -x^2 + 2x$, where h is any nonzero real number.

2.3 Question 4

What is the domain of the function $f(x)$?

$$f(x) = \sqrt{x^2 - 1}$$

Is this the same as the domain of the function $g(x)$?

$$g(x) = \sqrt{x^2 + 1}$$

2.4 Question 5

A soft drink company is designing a soda can. They want the soda can to be a cylinder that has a height that is four times the radius of the circular base.

- Write a function $V(h)$ for the volume enclosed by the soda can (in cubic inches) as a function of the height h of the can in inches.
- Write a function $S(r)$ for the surface area of the soda can (in square inches) as a function of the radius r of the circular base of the can in inches.

3 Problem Set 3

3.1 (*) Question 6

Consider the function

$$h(x) = \frac{2}{\sin^2(x)}$$

- What is the domain of $h(x)$? What is the range of $h(x)$?
- Is the function $h(x)$ even, odd, both even and odd, or neither?
- Sketch a graph of h .

3.2 (***) Question 7

Classify all functions f from \mathbb{R} to \mathbb{R} that have the following two properties:

- $g(x) = f(x) - 1$ is odd.
- $f(x)$ is even.