Qualifying Exam Syllabus

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August 28, 2020

Committee: Sunčica Čanić (Advisor), James Pitman, Daniel Tataru, Sung-Jin Oh (Chair)
Exam Date: Friday, August 28, 2020, 1-4 PM over Zoom

Major Topic: Partial Differential Equations (Analysis)

References: (1) L. C. Evans, *Partial Differential Equations*, (2) D. Tataru MAT 222A (Fall 2018) Notes

- *Fourier transform:* Definition of Fourier transform, basic properties, Schwartz space (Tataru 222A notes notes)
- *Distribution theory:* Definitions, homogeneous distributions, differentiation of distributions and multiplication by functions, convolutions, compactly supported distributions, tempered distributions, Fourier transform of tempered distributions (Tataru MAT 222A notes)
- *Basic linear PDEs:* Transport equation, Laplace's equation, heat equation, wave equation (Evans Chapter 2)
- *Nonlinear first order PDEs:* Characteristic ODEs, scalar conservation laws (Evans Chapter 3, except Section 3.2)
- *Sobolev spaces:* Hölder spaces, Sobolev spaces, approximation by smooth functions, extensions, trace operator, Sobolev inequalities, compactness, Poincare inequalities, difference quotients (Evans Chapter 5)
- *Elliptic PDE:* Lax-Milgram Theorem, existence, regularity, weak and strong maximum principles, eigenvalues of symmetric elliptic operators (Evans Chapter 6, except 6.5.2)
- *Parabolic and hyperbolic PDE:* Existence, regularity, maximum principles for parabolic PDE, hyperbolic systems, semigroup theory (Evans Chapter 7)
- *Calculus of Variations:* First and second variation and Euler-Lagrange equation, Euler-Lagrange equations for systems, existence of minimizers, regularity, constraints, Noether's theorem (Evans Chapter 8 except 8.5)
- *Systems of conservation laws:* Strict hyperbolicity, Rankine-Huginiot condition, genuine nonlinearity, rarefaction waves, contact discontinuities, shock waves, Riemann problem, Riemann invariants, entropy criteria, entropy-flux pairs (Evans Chapter 11)

Major Topic: Numerical Analysis of PDEs (Applied)

References: (1) R. J. LeVeque *Finite Difference Methods for ODEs and PDEs*, (2) Čanić MAT 228B (Spring 2020) Notes, (3) Čanić BIOENG C213 (Spring 2019) Notes

- *Finite differences:* Laplace's equation, heat equation, method of lines discretization, stability theory, von Neumann analysis, multi-dimensional problems, locally one-dimensional method (LeVeque 2, 3, 9)
- *Advection equations and systems of conservation laws:* Method of lines discretization, Lax-Wendroff, leapfrog, Lax-Friedrichs, upwind methods, von Neumann analysis, characteristics, CFL condition, modified equations, hyperbolic systems (LeVeque 10)
- *Application to numerical analysis of blood flow:* Derivation of reduced 1D model of balance laws describing blood flow, numerical solution of reduced 1D model of balance laws (Čanić BIOENG C213 (Spring 2019) Notes)
- *Finite elements:* Finite elements for elliptic problems, variational formulation, Galerkin formulation, error estimates, finite elements with higher order nodal basis functions (Čanić MAT 228B (Spring 2020) Notes)

Minor Topic: Probability Theory (Probability)

Reference: (1) R. Durrett *Probability: Theory and Examples*

- *Measure Theory:* Probability spaces, distributions, random variables, integration, expected value, product measures, Fubini's theorem, π-λ theorem (Durrett 1)
- *Law of Large Numbers:* Independence, weak law of large numbers, Borel-Cantelli lemmas, strong law of large numbers, convergence of random series (Durrett 2.1-2.5)
- *Characteristic Functions:* Weak convergence, characteristic functions (definition, inversion formula, weak convergence, moments and derivatives), Central Limit Theorems (Durrett 3.1, 3.2, 3.3.1-3.3.3, 3.4)
- *Martingales:* Conditional expectation, martingales, Doob's inequality, uniform integrability and convergence in L^1, backwards martingales, optional stopping theorems, applications to simple random walks, combinatorics of simple random walk (Durrett 4, except 4.5)
- *Markov chains with countable state space:* Markov property, Strong Markov property, recurrence and transience, stationary measures, asymptotic behavior (Chapter 5, except 5.4, 5.7, 5.8)
- *Brownian motion:* Definition, construction of Brownian motion, Markov property, Blumenthal's 0-1 Law, stopping times, strong Markov property, path properties, Brownian martingales (Chapter 7, except 7.6)