# Math 1A: Midterm 1 Review Problems

Jeffrey Kuan

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## Question 1

Find the domain of the function

$$f(x) = \frac{\ln(x^2 - 4)}{x^2}$$

## Question 2

Find the domain of the function

$$f(x) = \frac{x}{1 - \tan(x)}$$

Is this different from the domain of the function $g(x)$?

$$g(x) = \frac{\sqrt{x}}{1 - \tan(x)}$$

## Question 3

Compute, using the limit laws to justify your steps,

$$\lim_{x \to 2} \frac{\sqrt{x^2 + 5}}{\cos(\pi x)}$$

## Question 4

Calculate the following limit

$$\lim_{h \to 1} \frac{\sqrt{h^2 + h + 2} - 2}{h - 1}$$

## Question 5

Consider the two functions
$$f(x) = \frac{1}{x(x-4)}$$
$$g(x) = 2\,\cos(x)$$

Find $f \circ f$, $g \circ g$, $f \circ g$, and $g \circ f$, and find the domain of each of these functions.

## Question 6

Calculate the limit,
$$\lim_{x \to 0} \left( |x| + 3x + \sqrt{x} + 2 \right)$$

if it exists.

## Question 7

Compute the following quantities

$$\arccos \left( \cos \left( \frac{10\pi}{3} \right) \right)$$

$$\log_3 \left( \tan \left( \frac{7\pi}{6} \right) \right)$$

$$\sin \left( e^{\ln(\pi) - \ln(3)} \right)$$

## Question 8

Compute the following limit.

$$\lim_{x \to 0} \left[ \sqrt{x} \, \cos \left( \frac{1}{x} \right) + x \, \sin \left( \frac{1}{\sqrt{x}} \right) \right]$$

## Question 9

What value of the constant $a$ makes the following piecewise function continuous? Justify your answer.
$$f(x) = \frac{ae^x}{1+e^x} \qquad \text{if } x < 0$$
$$f(x) = \sqrt{x} \, \sin(x^2 - x) + 1 \qquad \text{if } x \geq 0$$

## Question 10

Compute the following limits.

$$\lim_{x \to \infty} \left[ \frac{2x^2 + 1}{3x^2 + 2x + \sqrt{x} + 1} \right]$$

$$\lim_{x \to -\infty} \left[ \frac{1}{\frac{\pi}{2} + \arctan(x)} \right]$$

$$\lim_{x \to 2} \left[ \frac{\cos(\pi x)}{(x-2)^5} \right]$$

## Question 11

Does the following limit exist?

$$\lim_{x \to 0} \frac{|x|}{x}$$

Now compare to this question. Does the following limit exist?

$$\lim_{x \to 0} \frac{|x| + 1}{x + 1}$$

## Question 12

Use an $\epsilon$-$\delta$ proof to show that the function $f(x) = x^2 + 1$ is continuous at $x = 0$.

## Question 13

Evaluate the following limit

$$\lim_{x \to 0} e^{\arctan(x) + (e^{3x}) - 1}$$

Justify your steps, by using the continuity of functions.

## Question 14

Consider the function

$$f(x) = \frac{1}{\sqrt{x}} \cos(x^2)$$

Does $\lim_{x \to 0^+} f(x)$ exist? If it does not exist, is $\lim_{x \to 0^+} f(x)$ infinity, negative infinity, or neither?

## Question 15

Let us consider the graph of $f(x) = \sqrt{x-2} + 3$. What transformations need to be applied to the graph of $f$ to get the graph of $g(x) = -2\sqrt{x+2}$?