

Math 1A: Midterm 2 Review Problems

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Skill Set 1: Derivatives

Question 1

Compute the derivative of

$$f(x) = \cos^2(e^{x+2})$$

Question 2

Compute the derivative of

$$f(x) = \frac{\tan(\ln(x))}{1 + \cos(\ln(x))}$$

Question 3

Compute the derivative of

$$f(x) = \arctan(\arcsin(2^{x+2}))$$

Question 4

Compute the derivative of

$$f(x) = 2^{\arccos(2^{\cos(x)})}$$

Question 5

Compute the second derivative of

$$f(x) = \sec^2(x)$$

Question 6

Compute the derivative of

$$f(x) = \arcsin(\sqrt{2 + \sin(e^x)})$$

Question 7

Compute the derivative of

$$f(x) = \sqrt[3]{\tan(4x) + \arcsin(2x)}$$

Question 8

Compute the derivative of

$$f(x) = e^{e^x \ln(\csc(x))}$$

Question 9

Compute the derivative of

$$f(x) = \log_2(\log_3(\log_4(x)))$$

Question 10

Compute the derivative of

$$f(x) = \frac{1}{\sqrt{\operatorname{arcsec}(\arcsin(x^5))}}$$

Skill Set 2: Logarithmic Differentiation

Question 1

Compute the derivative of

$$f(x) = x^{\sqrt{x}}$$

Question 2

Compute the derivative of

$$f(x) = (2x + 1)\sqrt{\sin^2(x)}$$

Question 3

Compute the derivative of

$$f(x) = (\ln(x))^{\ln(x)}$$

Question 4

Compute the derivative of

$$f(x) = (\arctan(x) + 2)^{\arctan(x)+2}$$

Question 5

Compute the derivative of

$$f(x) = x^{(x^2)} \cdot (2 + \cos(x))^{\sin(x)}$$

Skill Set 3: Implicit Differentiation

Question 1

Find an expression for $\frac{dy}{dx}$ for the following curve.

$$\sec(\sqrt{xy^2}) = \sin^2(x) + \cos^2(y)$$

Question 2

Find the equation of the tangent line and normal line to the following curve at $(1, 1)$.

$$\ln(\sqrt{xy}) = x^2y^3 - e^{x-1}\sqrt{y}$$

Question 3

Find an expression for $\frac{dy}{dx}$ for the following curve.

$$\arctan(e^{\sqrt{xy}}) = x^2e^y + y^3$$

Question 4

Find the equation of the tangent line and normal line to the following curve at $(1, 1)$.

$$\sin(x^3y^3 - 1) = e^{(e^{y-1}-1)} - (xy)^{4/3}$$

Question 5

Find the equation of all tangent lines to the ellipse

$$x^2 + 3y^2 = 12$$

that pass through the point $(0, 4)$. (Careful, this is not asking for the tangent line to $(0, 4)$, because as you can check, the point $(0, 4)$ is not actually on the ellipse.)

Skill Set 4: Curve Sketching

Question 1

Sketch the graph of the curve

$$f(x) = \frac{x^3}{x^3 + x}$$

and label all zeros, asymptotes, local maxima, local minima, and points of inflection.

Question 2

Sketch the graph of the curve

$$f(x) = \arctan(e^x)$$

and label all zeros, local maxima, local minima, and points of inflection.

Question 3

Sketch the graph of the curve

$$f(x) = x^{1/4} - 4x^{-3/4}$$

and label all zeros, local maxima, local minima, and points of inflection.

Question 4

Sketch the graph of the curve

$$f(x) = \sqrt{x} \cdot \ln(x)$$

and label all zeros, local maxima, local minima, and points of inflection.

Question 5

Sketch the graph of the curve

$$f(x) = \frac{x^4}{x^3 + x}$$

and label all zeros, asymptotes, local maxima, local minima, and points of inflection.

Skill Set 5: Optimization

Question 1

Consider the circle $x^2 + y^2 = 1$. What is the maximum area of a triangle with vertices at $(1, 0)$, (x, y) , and $(-x, y)$, where (x, y) is on the circle? (Caution! Note that we are not assuming that y is nonnegative.)

Question 2

A triangle has two sides of length 1. What is the maximum area of the triangle?

Question 3

What is the smallest distance between a point on the graph of $y = \frac{2}{\sqrt{x-1}}$ and the point $(1, 0)$?

Question 4

What is the largest area of a rectangle inscribed in the ellipse $9x^2 + 16y^2 = 1$, where the rectangle is centered at the origin?

Question 5

Find the shortest distance from the curve $x^4 - 3y^2 = -2$ to the origin.

Question 6

Find the shortest distance from the point $(1, 2)$ to the circle $x^2 + y^2 = 20$.

Skill Set 6: Mean Value Theorem, Intermediate Value Theorem, Rolle's Theorem

Question 1

Show that the equation $\sin(4x) = -6x + 1$ has exactly one solution.

Question 2

Show that the function $\cos(x) + \arctan(x) + 2x - 1$ has exactly one zero.

Question 3

Show that the equation $\cos^2(x) + x = e^{-x}$ has exactly one solution.

Question 4

Show that the equation $x^5 = \frac{10}{3}x^3 - 5x + 3$ has exactly one solution.

Question 5

Show that the equation $x^4 + 2x^2 = 10$ has exactly two solutions.

Skill Set 7: L'Hopital's Rule

Question 1

Compute the following limit.

$$\lim_{x \rightarrow 2} \frac{x^3 - 3x - 2}{x^2 - 3x + 2}$$

Question 2

Compute the following limit.

$$\lim_{x \rightarrow 0^+} (x^3 + 1)^{1/x}$$

Question 3

Compute the following limit.

$$\lim_{x \rightarrow \infty} x^2 \ln(x)$$

Question 4

Compute the following limit.

$$\lim_{x \rightarrow \infty} \frac{\arctan(x) \ln(x)}{x^2}$$

Question 5

Compute the following limit.

$$\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x^2}\right)^{\sqrt{x}}$$

Question 6

Compute the following limit.

$$\lim_{x \rightarrow 0^+} (\sqrt{x})^{x^2}$$

Question 7

Compute the following limit.

$$\lim_{x \rightarrow \infty} (1 + e^{-x})e^x$$