Math 54 Midterm 1 (Practice Exam 4)

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Instructions:

- This exam is **110 minutes** long.
- No calculators, computers, cell phones, textbooks, notes, or cheat sheets are allowed.
- All answers must be justified. Unjustified answers will be given little or no credit.
- You may write on the back of pages or on the blank page at the end of the exam. No extra pages can be attached.
- There are 5 questions.
- Question 1 is 40 points, Question 2 is 50 points, and Questions 3, 4, and 5 are 20 points each, for a total of **150 points**.
- Good luck!

Problem 1

Give an example of each of the following, or briefly explain why the object described cannot exist. [5 pts each]

Part (a)

Three 3×3 matrices that solve $A^2 = I$.

Part (b)

Three 2×2 matrices with all positive entries and determinant 1.

Part (c)

A vector $\mathbf{v}$ in $\mathbb{R}^3$ such that $(1, 2, 1)$, $(-2, -4, -2)$, and $\mathbf{v}$ are linearly independent in $\mathbb{R}^3$.

Part (d)

A coefficient matrix A such that $A\mathbf{x} = 0$ does not have a unique solution, and for some $\mathbf{b}$, the system $A\mathbf{x} = \mathbf{b}$ is inconsistent.

Part (e)

Two matrices A and B such that $\operatorname{tr}(AB) \neq \operatorname{tr}(A)\operatorname{tr}(B)$.

Part (f)

A 2×2 matrix A such that $\det(A^{-1}) = \frac{1}{4}$.

Part (g)

An invertible 3 by 3 matrix with all entries positive.

Part (h)

A 3×3 invertible matrix where the second row is four times the first row.

Problem 2

Short answer. [10 points each]

Part (a)

Consider the matrices

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Calculate A^{-1}, and calculate $(A^{-1}BA)^{101}$.

Part (b)

If A is a square matrix such that $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$, what is the reduced row echelon form of A? Is A invertible?

Part (c)

Find all λ such that the following matrix is invertible.

$$\begin{bmatrix} 0 & 6/5 & 1 \\ \lambda & \lambda & 1 \\ 5 & \lambda & 0 \end{bmatrix}$$

Part (d)

Find values of c and d such that the following system has (a) no solution, (b) exactly one solution, and (c) infinitely many solutions.

$$x_1 + x_2 = c$$

$$3x_1 + dx_2 = 2d$$

Part (e)

Let us consider the following matrices.

$$A = \begin{bmatrix} 12 & 11 & 10 & 9 & 8 \\ 7 & 6 & 5 & 4 & 3 \\ 2 & 1 & 0 & -1 & -2 \\ -3 & -4 & -5 & -6 & -7 \\ -8 & -9 & -10 & -11 & -12 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find $\det(A^{2019}B^{2019})$.

Problem 3

Part (a)

Consider the following **elementary matrix**

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Calculate AB, where $B = \begin{bmatrix} 1 & 0 & 2 & 1 \\ -1 & 2 & 0 & 1 \\ 0 & -1 & -1 & 3 \\ 4 & 2 & 1 & 1 \end{bmatrix}$. [8 points]

Part (b)

Multiplying a matrix B on the left by the **elementary matrix** A has the same effect as performing a certain elementary row operation on B. Which elementary row operation is it? [4 points]

Part (c)

What is A^{-1}? [8 points]

Problem 4

Find conditions on a, b, c, and d such that the vectors

$$\mathbf{v_1} = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

are linearly independent in $\mathbb{R}^4$. [20 points]

Problem 5

Find all x_1, x_2, x_3, x_4 such that

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

If you solve this using a system of linear equations, solve the system of equations using row reduction. [20 points]