Math 54 Midterm 1 (Practice Exam 2)

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Instructions:

- This exam is **110 minutes** long.
- No calculators, computers, cell phones, textbooks, notes, or cheat sheets are allowed.
- All answers must be justified. Unjustified answers will be given little or no credit.
- You may write on the back of pages or on the blank page at the end of the exam. No extra pages can be attached.
- There are 5 questions.
- Question 1 is 40 points, Question 2 is 50 points, and Questions 3, 4, and 5 are 20 points each, for a total of **150 points**.
- Good luck!

Problem 1

Give an example of each of the following, or briefly explain why the object described cannot exist. [5 pts each]

Part (a)

Two row equivalent matrices A and B such that $A\mathbf{x} = 0$ and $B\mathbf{x} = 0$ have different solution sets.

Part (b)

A nonzero 4×4 matrix A such that $A = A^{-1}$.

Part (c)

A set of vectors that span $\mathbb{R}^2$ but are not linearly independent.

Part (d)

A symmetric 4 by 4 matrix with determinant -20.

Part (e)

A system of equations $A\mathbf{x} = \mathbf{b}$ in three variables x_1, x_2, x_3 with $\mathbf{b}$ nonzero, whose solution set is a line through the origin.

Part (f)

A matrix that commutes with every matrix that is not the zero matrix or the identity matrix.

Part (g)

A square matrix whose columns are linearly independent, but whose rows are not linearly independent.

Part (h)

A nonzero 3 by 3 matrix with trace zero and determinant zero.

Problem 2

Short answer. [10 points each]

Part (a)

Calculate the determinant of the following matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 2 \end{bmatrix}$$

Part (b)

Classify all values x, y, and z so that $A\mathbf{x} = \mathbf{b}$ is consistent for every vector $\mathbf{b}$.

$$A = \begin{bmatrix} 0 & 1 & 3 & z & -2 \\ 0 & 0 & 0 & y & 1 \\ 0 & 0 & 0 & x & 0 \end{bmatrix}$$

Part (c)

Let A be the invertible matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, and let D be the diagonal matrix $D = \begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix}$.

Find a matrix M such that $M^2 = A^{-1}DA$. Express your answer as a single 2 by 2 matrix.

Part (d)

Suppose that for some coefficient matrix A, $A\mathbf{x} = 0$ has a unique solution. Does $A\mathbf{x} = \mathbf{b}$ necessarily have a unique solution for every $\mathbf{b}$?

Part (e)

Define

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 \\ 4 & 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 & 1 \end{bmatrix}$$

Find $\det(B^4 A)$.

Problem 3

Show that the columns of the following matrix are linearly independent and span $\mathbb{R}^6$.

$$M = \begin{bmatrix} 3 & 2 & 4 & 0 & 4 & 0 \\ 2 & 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 \\ 7 & 3 & 4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 2 & 3 & 4 & 1 & 1 \end{bmatrix}$$

[20 points]

Problem 4

Find all x_1, x_2, x_3, and x_4 that solve the following matrix equation.

$$\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}$$

If you use a system of equations to solve this, solve the system of equations by row reducing to reduced row-echelon form. [20 points]

Problem 5

Part (a)

Calculate the inverse of the following upper triangular matrix. [10 points]

$$M = \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Part (b)

Show, **without using the determinant**, that every upper triangular matrix with nonzero diagonal entries is invertible. [10 points]