Math 54 Quiz 6

October 17, 2019

Directions: Answer all questions below. This quiz is out of 15 points, even though there are 16 points total. (There is a built-in one point curve.)

Question 1 (5 points)

Directions: For each item, circle either True or False. (0.5 points each)

- (True/False) If v is an eigenvector of two matrices A and B, then v is also an eigenvector of $A + B$. (Hint: Carefully apply the definition.)
- (True/False) Every real-valued upper triangular matrix is diagonalizable.
- (True/False) Every real-valued matrix is diagonalizable over the complex numbers.
- (True/False) If v is an eigenvector of A with eigenvalue λ, then $2v$ is an eigenvector of A with eigenvalue 2λ.
- (True/False) A 2019 by 2019 real-valued matrix has at least one real eigenvalue.
- (True/False) Any 5 by 5 real-valued matrix that has $1 + 2i$ and $-1 - 3i$ as eigenvalues is diagonalizable.
- (True/False) Any diagonalizable real-valued matrix A whose only eigenvalue is 1 must be the identity matrix. (Hint: What would SDS^{-1} look like for such a matrix?)
- (True/False) Any noninvertible real-valued matrix has at least one real eigenvalue.
- (True/False) If A and B are diagonalizable, then so is $A + B$.
- (True/False) If an invertible matrix A is diagonalizable, then so is A^{-1}. (Hint: Think about SDS^{-1}. Is 0 an eigenvalue of A?)

Question 2 (6 points)

Consider the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

- Find all eigenvalues of A and their algebraic and geometric multiplicities. Find a basis for each eigenspace of A.
- Find all real numbers λ such that $\lim_{n\to\infty} \lambda^n A^n$ exists.

Question 3 (5 points)

Find all real numbers k such that

$$B = \begin{bmatrix} 0 & -1 \\ 4 & k \end{bmatrix}$$

is diagonalizable over the real numbers. Then, find all real numbers k such that B is diagonalizable over the complex numbers.

(Hint: Think carefully about what needs to be done here, so that you can avoid doing any unnecessary calculations. If done in the most efficient way, this question should not take very long.)