Math 54 Quiz 4

September 19, 2019

Question 1 (2 points)

Directions: For each item, circle either True or False. (0.5 points each)

- (True/False) There is a surjective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^4$.
- (True/False) Every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ is an isomorphism.
- (True/False) If A is an $n \times n$ matrix, then if $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^n$, then $A\mathbf{x} = \mathbf{0}$ has a unique solution.
- (True/False) If A and B are both invertible square matrices of the same size, then $A + B$ is also invertible.

Question 2 (6 points)

Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that

$$T(1,1) = (1,1,1) \qquad T(2,-1) = (0,1,-1)$$

Determine if T is injective, surjective, bijective.

Question 3 (7 points)

Define the cyclic permutation linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ by

$$T(x_1, x_2, x_3, x_4) = (x_4, x_1, x_2, x_3)$$

- Find the matrix A for T.
- Is T injective, surjective, bijective? (Hint: Your row reduction can easily be done by just switching rows and using no other elementary row operations.)
- If T is bijective, find a formula for $T^{-1}(y)$, where $y = (y_1, y_2, y_3, y_4)$.