Math 54 Practice Quiz 5

September 25, 2019

Part 1: True/False (15 points)

Directions: For each item, circle either True or False. (1 point each)

- (True/False) The span of the vectors $(1, 2, 1)$, $(1, -1, 0)$, and $(1, 0, 0)$ has exactly three vectors in it.
- (True/False) $1, x, x^2$ are linearly independent in P_{10} (the space of polynomials with real coefficients with degree less than or equal to 10).
- (True/False) The column space of the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is a subspace of $\mathbb{R}^2$.
- (True/False) Three linearly independent vectors in $\mathbb{R}^3$ form a basis for $\mathbb{R}^3$.
- (True/False) The dimension of P_{10} is 10.
- (True/False) The dimension of $M_{m\times n}$, the space of m by n matrices with real entries, is $m + n$.
- (True/False) The set of vectors in $\mathbb{R}^4$ satisfying $x_1 - x_2 + 2x_3 - x_4 = 1$ is a subspace.
- (True/False) The set of polynomials in P_3 such that $p'(2) = 0$ is a subspace.
- (True/False) The set of 3 by 3 matrices with integer entries is a subspace of $M_{3\times 3}$.
- (True/False) The left nullspace of an m by n matrix is a subspace of $\mathbb{R}^n$.
- (True/False) The matrices $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ form a basis for $M_{2\times 2}$.
- (True/False) Every basis for a vector space contains the same number of elements.
- (True/False) A square matrix whose reduced row echelon form has a pivot in every column is invertible.
- (True/False) There is a 2 by 3 matrix with a row space that is all of $\mathbb{R}^3$.
- (True/False) $AB = BA$ for any two square matrices A and B of the same size.

Part 2: Short Answer (15 points)

Directions: Provide a short answer for each question (no work needed). The questions here should not be computationally difficult. (In particular, no involved calculations are needed.)

Short Answer 1 (6 points)

Let $A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. (1.5 points each)

- Find three distinct vectors in the row space of A.
- Find three distinct vectors in the nullspace of A.
- Find three distinct vectors in the column space of A.
- Find three distinct vectors in the left nullspace of A. (This can be done easily without calculating anything.)

Short Answer 2 (4.5 points)

Consider the vector space $M_{2\times 3}$, the space of all 2 by 3 real valued matrices. (1.5 points each)

- Find eight vectors that span $M_{2\times 3}$, or explain why such vectors cannot exist.
- Find three linearly independent vectors in $M_{2\times 3}$, or explain why such vectors cannot exist.
- Find a basis for $M_{2\times 3}$. What is the dimension of $M_{2\times 3}$?

Short Answer 3 (4.5 points)

Consider the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \end{bmatrix}$. (1.5 points each)

- Find a basis for the left nullspace. What is its dimension?
- Find a basis for the row space. What is its dimension?
- Calculate AA^t.