Math 54: Problem Set 1

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This problem set is due **Friday, June 28, 2019, 4 PM**. You may collaborate with other students in the class on this problem set, but your solutions must be own, and you should write up the solutions to the problems by yourself. Some of the problems below may be challenging, so start early, and feel free to come to office hours or ask questions on Piazza if you need help.

Textbook Problems

These questions are from Lay, Lay, McDonald *Linear Algebra and Its Applications*

- **Section 1.1:** 11, 14, 18, 21, 23, 24
- **Section 1.2:** 3, 12, 13, 20, 23, 29, 30, 31
- **Section 1.3:** 11, 18, 21, 22, 24, 25
- **Section 1.5:** 7, 12, 21, 23, 26, 28, 30, 32

Additional Problems

Problem 1

As we discussed in lecture, linearity is an important property in many linear algebra problems. In this question, we will introduce this idea, which we will also explore later in the course.

- Consider the system of equations

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 0 \\ x_1 - 2x_2 - 2x_3 &= 0 \\ x_1 + 5x_2 + 3x_3 &= 0 \end{aligned}$$

  Show that if (r_1, r_2, r_3) and (s_1, s_2, s_3) are two solutions of this system, then for any two real numbers c_1 and c_2,

$$c_1(r_1, r_2, r_3) + c_2(s_1, s_2, s_3) = (c_1 r_1 + c_2 s_1, c_1 r_2 + c_2 s_2, c_1 r_3 + c_2 s_3)$$

  is also a solution.

- Now consider the general situation. Consider a general system of equations with m equations and n variables (m is not necessarily equal to n)

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = 0$$

$$...$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = 0$$

where all of the a_{ij} are real numbers (they are the coefficients in the system of equations). Show that if $(r_1, r_2, ..., r_n)$ and $(s_1, s_2, ..., s_n)$ are two solutions of this system, then for any two real numbers c_1 and c_2,

$$c_1(r_1, r_2, ..., r_n) + c_2(s_1, s_2, ..., s_n) = (c_1r_1 + c_2s_1, c_1r_2 + c_2s_2, ..., c_1r_n + c_2s_n)$$

is also a solution.