# Math 1B Final Review 4

April 30, 2019

**Topics to Review:**

- Power series (radius/interval of convergence)
- Taylor series

## Question 1

Find the interval of convergence of the following power series.

$$\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{3n}{n+2}\right)^n x^n$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\ln(n)}{\sqrt{n}}(2x-3)^n$$

Find the radius of convergence of the following power series.

$$\sum_{n=1}^{\infty}\frac{n!}{n^n}(x-1)^n$$

$$\sum_{n=1}^{\infty}\frac{(2n)!}{(2n)^{2n}}x^{2n}$$

$$\sum_{n=1}^{\infty}\frac{n!\cdot 2\cdot 5\cdot \ldots\cdot(3n-1)}{(2n)!}x^{2n}$$

## Question 2

Taylor expand the following functions around $x=0$ (so find a Maclaurin expansion) and find a general formula for $f^{(n)}(0)$.

$$f(x)=\frac{x+2}{e^x}$$

$$f(x)=\frac{2x}{1+x^4}$$

$$f(x)=\int_0^x \sin(t^2)dt$$

# Question 3

Taylor expand the following function around $x = 2$ and find a formula for $f^{(n)}(2)$.

$$f(x) = (x-2)\cos(x^2 - 4x + 4)$$

# Question 4

Prove Euler's formula $e^{ix} = \cos(x) + i\sin(x)$ using the following steps.

- Find the Maclaurin expansion for $e^{ix}$.
- Find the Maclaurin expansion for $\cos(x) + i\sin(x)$.
- Check that these expansions are equal. (You may need to shift indices and simplify to see this).