Math 1B: Integration Nation

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Team Name: _____

Team Member Names: _____

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Welcome to America's favorite integration-based game show, Integration Nation! You and your team will have to tackle 20 of the gnarliest integrals out there. Here are the rules.

- In each question, you will calculate the derivative of the given function, for the number of points specified (6 points, 12 points).

- You will be given a score based on the following rubric.

  - Full (100%) credit: Integral is calculated correctly, or almost completely correctly.
  - Almost complete (75%) credit: Integral is calculated with a few minor mistakes.
  - Half (50%) credit: Integral is calculated with several mistakes.
  - Almost no (25%) credit: Integral is calculated with major mistakes.
  - No (0%) credit: No significant attempt.

- Some questions also have a bonus question.

  - If you answer the bonus question correctly, you get 1.5 times as many points for that question, but ONLY if you got full credit for that integral.
  - So if Question X is worth 6 points, if you answer Question X and its bonus correctly, you will get a total of 9 points.
  - But if you do not get full credit for the integral in Question X, even if you get the bonus question for Question X right, you will get no points for the bonus.

- The maximum possible score on the questions is 225 points.

- Integrals are hard. So you will get 75 extra points that you can use to buy hints, which will be provided for certain "prices" on the board. Hints are most expensive for the first team that buys them for a certain question.

- Good luck and have fun!

Question 1 (6 points)

$$\int \frac{\arctan(x)e^{-\arctan(x)}}{1+x^2}dx$$

Question 2 (6 points)

$$\int \frac{2x+1}{x^2-4x+5}dx$$

Bonus: Compute

$$\int_1^\infty \frac{2x+1}{x^2-4x+5}dx$$

Question 3 (6 points)

$$\int \frac{x}{\sqrt{x^4 - 2}} dx$$

Question 4 (6 points)

$$\int e^x \sec^5(e^x)\tan^5(e^x) dx$$

Bonus: What is the domain of the integrand $f(x) = e^x \sec^5(e^x)\tan^5(e^x)$?

Question 5 (6 points)

$$\int e^{-4x} \sin(2x)\, dx$$

Bonus: Find

$$\int_0^\infty e^{-4x} \sin(2x)\, dx$$

Question 6 (6 points)

$$\int \frac{1+x^2}{\sqrt{1-x^2}}\, dx$$

Bonus: Calculate T_2 (trapezoidal rule with $n = 2$) for the definite integral

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{1+x^2}{\sqrt{1-x^2}}\, dx$$

Question 7 (6 points)

$$\int \tan^3(x)\,dx$$

Bonus: Compute

$$\int_0^{\frac{\pi}{2}} \tan^3(x)\,dx$$

Question 8 (6 points)

$$\int (x^4 + x^2)\arctan(x)\,dx$$

Question 9 (6 points)

$$\int \frac{1}{(x^2 + 3)^{5/2}} dx$$

Question 10 (6 points)

$$\int \ln(x) \left(1 + \frac{1}{x} + \frac{1}{x^2}\right) dx$$

Question 11 (12 points)

$$\int (1 + \sec(x) + \sec^2(x) + \sec^3(x) + \sec^4(x))dx$$

Bonus: Consider the definite integral

$$I = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} (1 + \sec(x) + \sec^2(x) + \sec^3(x) + \sec^4(x))dx$$

and let R_4 be the right endpoint approximation to this definite integral with $n = 4$. Is R_4 greater than, less than, or equal to I? Justify your answer.

Question 12 (12 points)

$$\int \frac{x^{1/3} + 1}{x^{2/3}(x + 1)}dx$$

Question 13 (12 points)

$$\int \frac{3x^4 - 2x^3 + 14x^2 - 11x + 1}{x^3 - x^2 + 4x - 4} dx$$

Bonus: Find the vertical/horizontal/slant asymptotes of $f(x) = \frac{3x^4 - 2x^3 + 14x^2 - 11x + 1}{x^3 - x^2 + 4x - 4}$.

Question 14 (12 points)

$$\int \sin(\ln(x)) dx$$

Bonus: Compute

$$\int_1^\infty \sin(\ln(x)) dx$$

Question 15 (12 points)

$$\int \sin(2x)\sin^2(x)\sqrt{1+\sin^2(x)}\,dx$$

Question 16 (12 points)

$$\int \frac{\sqrt{x}}{1+x}\,dx$$

Bonus: Find the midpoint approximation M_2 $(n=2)$ for the definite integral

$$\int_{3/2}^{23/2} \frac{\sqrt{x}}{1+x}\,dx$$

Question 17 (12 points)

$$\int \frac{e^{2x} + e^{-2x}}{\sqrt{e^{4x} - 4}} dx$$

Question 18 (12 points)

$$\int \frac{x^3 + 3x^2 + 8x + 1}{(x^2 + 2x + 3)^2} dx$$

Bonus: Find the vertical/horizontal/slant asymptotes of $f(x) = \frac{x^3 + 3x^2 + 8x + 1}{(x^2 + 2x + 3)^2}$.

Question 19 (12 points)

$$\int (\sin^4(x) + 2\sin^2(x)\cos^2(x) + \cos^4(x))^4 - 1 dx$$

Bonus: What is the range of the function

$$f(x) = (\sin^4(x) + 2\sin^2(x)\cos^2(x) + \cos^4(x))^4 - 1$$

Question 20 (12 points)

$$\int \sin(x)\sin(4x) + \arcsin(x) + \arcsin(4x)dx$$