Math 54 Final Exam (Practice 3)

Jeffrey Kuan

August 16, 2019

Name: ____

SSID: ____

Instructions:

- This exam is **110 minutes** long.
- No calculators, computers, cell phones, textbooks, notes, or cheat sheets are allowed.
- All answers must be justified. Unjustified answers will be given little or no credit.
- You may write on the back of pages or on the blank page at the end of the exam. No extra pages can be attached.
- There are 7 questions.
- The exam has a total of **200 points**.
- Good luck!

Problem 1 (30 points)

Part (a)

Let U be the subset of 3 by 3 matrices with integer entries. Is U a subspace of $M_{3\times 3}$? Prove your answer. [10 points]

Part (b)

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the translation map, $T(x, y, z) = (x + 1, y + 1, z + 1)$. Is T a linear transformation? Prove your answer. [10 points]

Part (c)

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the matrix multiplication map given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ 0 & -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$

Is T an isometry? Prove your answer. [10 points]

Problem 2 (30 points)

Part (a)

Show that there is no one-to-one linear transformation T from $M_{2\times 2}$ to P_2. [15 points]

Part (b)

Is there a bijective linear transformation T from $M_{2\times 2}$ to P_3? Either prove that one does not exist, or find an example of one. [15 points]

Problem 3 (30 points)

Consider the linear transformation $T : P_4 \to P_4$ given by

$$T(p(x)) = \frac{d^2}{dx^2}((x-1)p(x))$$

Find the trace, determinant, and characteristic polynomial of T. Find all eigenvalues and eigenvectors of T. Is T diagonalizable?

Problem 4 (30 points)

Consider the subspace W of $\mathbb{R}^4$ spanned by $(1,2,1,0)$ and $(0,1,1,1)$.

Part (a)

Find an orthonormal basis for W. [10 points]

Part (b)

Find an orthonormal basis for $W^\perp$. [20 points]

Problem 5 (30 points)

Find a particular solution to the differential equation

$$y'' - 3y' + 2y = 2xe^{2x} - x$$

such that $y(0) = 1$ and $y'(0) = 0$.

Problem 6 (30 points)

Find the Fourier series expansion for

$$f(\theta) = -1, \quad -\pi < x < 0 \qquad\qquad f(\theta) = 1, \quad 0 < x < \pi$$

Write the expansion as both an infinite sum of complex exponentials, and also as an infinite sum of sines and cosines.

Problem 7 (20 points)

Match the systems of differential equations to the appropriate phase portrait **on the next page**. Note that not every figure on the next page will be used[1].

$$\begin{aligned} x_1'(t) &= x_2(t) \\ x_2'(t) &= -x_1(t) \end{aligned} \tag{1}$$

$$\begin{aligned} x_1'(t) &= 2x_1(t) + x_2(t) \\ x_2'(t) &= -x_1(t) + 2x_2(t) \end{aligned} \tag{2}$$

$$\begin{aligned} x_1'(t) &= -x_1(t) \\ x_2'(t) &= -x_2(t) \end{aligned} \tag{3}$$

$$\begin{aligned} x_1'(t) &= x_1(t) + 2x_2(t) \\ x_2'(t) &= 2x_1(t) + x_2(t) \end{aligned} \tag{4}$$

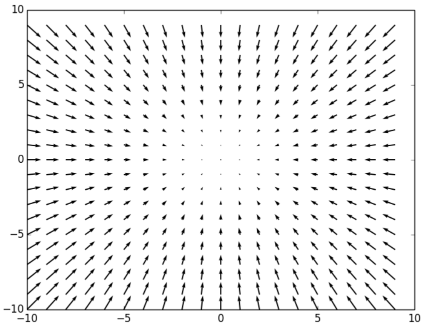
$$\begin{aligned} x_1'(t) &= -3x_1(t) + x_2(t) \\ x_2'(t) &= -x_1(t) - 3x_2(t) \end{aligned} \tag{5}$$

$$\begin{aligned} x_1'(t) &= -x_1(t) + x_2(t) \\ x_2'(t) &= -x_1(t) - 3x_2(t) \end{aligned} \tag{6}$$

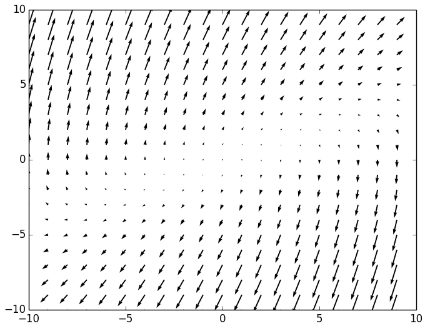
END OF EXAM

(Figures on next page)

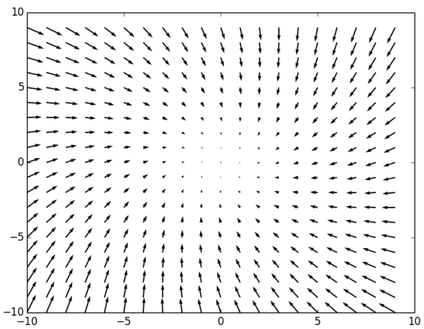
[1]Figures generated using Matplotlib



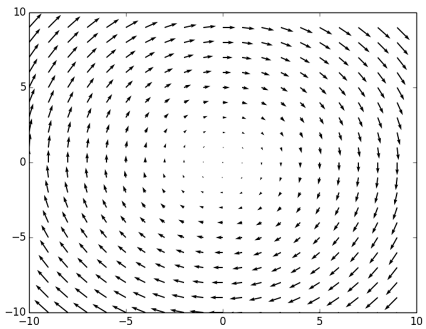
(a) Graph (A)



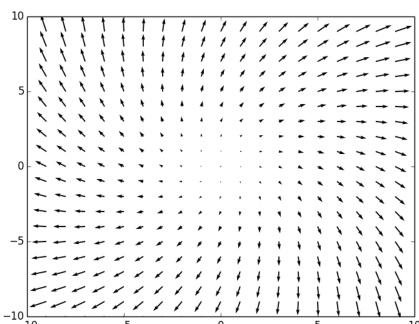
(b) Graph (B)



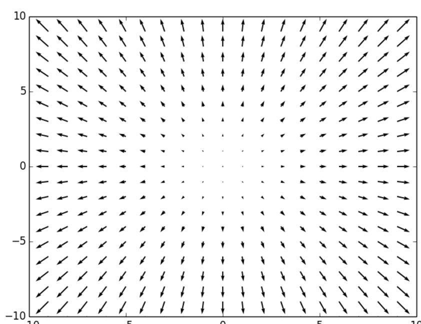
(c) Graph (C)



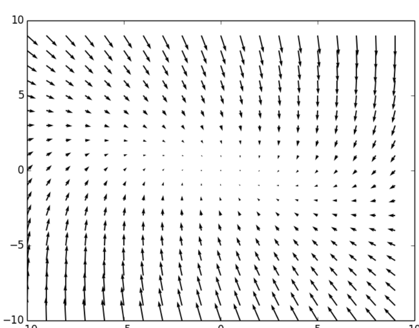
(d) Graph (D)



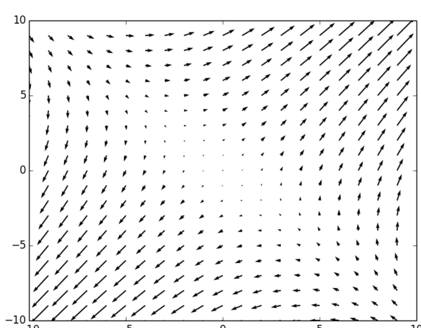
(e) Graph (E)



(f) Graph (F)



(g) Graph (G)



(h) Graph (H)