Math 54 Final Exam (Practice 2)

Jeffrey Kuan

August 16, 2019

Name: Answer Key

SSID: ______________________________

Instructions:

- This exam is **110 minutes** long.
- No calculators, computers, cell phones, textbooks, notes, or cheat sheets are allowed.
- All answers must be justified. Unjustified answers will be given little or no credit.
- You may write on the back of pages or on the blank page at the end of the exam. No extra pages can be attached.
- There are 7 questions.
- The exam has a total of **200 points**.
- Good luck!

Problem 1 (30 points)

Let S be the set of 2 by 2 matrices that commute with the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$.

Part (a)

Show that S is a subspace of $M_{2\times 2}$. [10 points]

If B and C commute with A, so does $c_1 B + c_2 C$.

$$\begin{aligned}(c_1 B + c_2 C)A &= c_1 BA + c_2 CA \\ &= c_1 AB + c_2 AC \\ &= A(c_1 B + c_2 C) \checkmark\end{aligned}$$

So S is a subspace of $M_{2\times 2}$.

Part (b)

Find a basis for S. [20 points]

$$\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - x_2 & 2x_1 \\ x_3 - x_4 & 2x_3 \end{bmatrix} - \begin{bmatrix} x_1 + 2x_3 & x_2 + 2x_4 \\ -x_1 & -x_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -x_2 - 2x_3 & 2x_1 - x_2 - 2x_4 \\ x_1 + x_3 - x_4 & x_2 + 2x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & -2 & 0 \\ 2 & -1 & 0 & -2 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 \end{bmatrix} \to \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 \end{bmatrix} \xrightarrow{R_2 - 2R_3} \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_3 \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\boxed{\text{Basis: } \begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

Problem 2 (30 points)

Consider

$$\mathcal{B} = \left\{ \underset{M_1}{\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}}, \underset{M_2}{\begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}}, \underset{M_3}{\begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}} \right\}$$

Part (a)

Show that $\mathcal{B}$ is a basis for the vector space of skew-symmetric 3 by 3 matrices, $\text{Skew}_{3\times 3}$. [10 points]

Show lin. ind: $c_1\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + c_2\begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} + c_3\begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{aligned} c_1 + 2c_2 \quad &= 0 \\ -c_1 \quad - 2c_3 &= 0 \\ c_2 \quad &= 0 \end{aligned}$$

$\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix}$ has det $= 2 \neq 0$

so only has trivial sln ✓.

Show span: Check that

$c_1\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + c_2\begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} + c_3\begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

always has a solution.

$$\begin{aligned} c_1 + 2c_2 \quad &= a \\ -c_1 \quad -2c_3 &= b \\ c_2 \quad &= c \end{aligned}$$

Since $\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix}$ has det $= 2 \neq 0$, there is always a soln by Invertible Matrix Theorem. ✓

Part (b)

For $t : \text{Skew}_{3\times 3} \to \text{Skew}_{3\times 3}$, calculate $[t]_{\mathcal{B}\to\mathcal{B}}$, where t is the matrix transpose linear transformation that sends A to A^t. [20 points]

$$t\left(\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} = -M_1$$

$t(M_2) = -M_2 \qquad t(M_3) = -M_3$

$$\boxed{[t]_{\mathcal{B}\to\mathcal{B}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}$$

Problem 3 (30 points)

Find all least squares solutions to $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & -1 & 3 & 1 \\ 1 & 1 & 0 & -1 & -1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 0 & -1 & -1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & 0 & -2 & -1 & 3 & -1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ span Col(A).

ONB for Col(A): $w_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$$v_2 - \langle v_2, w_1 \rangle w_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{2}(3)\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}$$

$$w_2 = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$w_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad w_2 = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\text{proj}_{\text{Col}(A)} \vec{b} = \langle b, w_1 \rangle w_1 + \langle b, w_2 \rangle w_2$$

$$= \frac{1}{2}(2)\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \frac{1}{6}(-4)\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{3} \\ \frac{5}{3} \\ -\frac{4}{3} \end{pmatrix} \qquad A\hat{x} = \text{proj}_{\text{Col}(A)} \vec{b}$$

$$\left[\begin{array}{ccccccc|c} 1 & 2 & 0 & 0 & -1 & 3 & 1 & \frac{1}{3} \\ 1 & 1 & 0 & -1 & -1 & 3 & 0 & \frac{5}{3} \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & -\frac{4}{3} \end{array}\right] \to \left[\begin{array}{ccccccc|c} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 & -1 & 3 & -1 & 3 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & -\frac{4}{3} \end{array}\right] \begin{array}{l} R_1 - R_2 - R_3 \\ R_2 - R_3 \\ \\ \end{array}$$

$$\begin{array}{ccccccc} & & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \end{array}$$

free

$$\boxed{\hat{x} = \begin{pmatrix} 3 \\ -\frac{4}{3} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_3\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4\begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_6\begin{pmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_7\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}$$

Problem 4 (30 points)

Consider the matrix multiplication linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$, given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 2 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Part (a)

Find a basis for ker(T) and range(T). [10 points]

$\ker(T) = \text{null space}(A)$

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 2 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\uparrow\uparrow$ pivot cols

$$\boxed{\text{Basis for } \ker(T) = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}$$

$$\boxed{\text{Basis for range}(T) = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 2 \\ 1 \end{pmatrix}}$$

Part (b)

Determine if the point $\underset{}{\overset{v=}{(1, 3, 4, 5)}}$ is in range(T). If not, find the distance from $(1, 3, 4, 5)$ to range(T) in $\mathbb{R}^4$. [20 points]

$$w_1 = \frac{1}{\sqrt{7}}\begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix} \qquad v_2 - \langle v_2, w_1 \rangle w_1 = \begin{pmatrix} -1 \\ 0 \\ 2 \\ 1 \end{pmatrix} - \frac{1}{7}(-2)\begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$$

$$w_2 = \frac{1}{\sqrt{266}}\begin{pmatrix} -5 \\ 4 \\ 12 \\ 9 \end{pmatrix} \qquad = \begin{pmatrix} -\frac{5}{7} \\ \frac{4}{7} \\ \frac{12}{7} \\ \frac{9}{7} \end{pmatrix} \Rightarrow \begin{pmatrix} -5 \\ 4 \\ 12 \\ 9 \end{pmatrix}$$

$25 + 16 + 144 + 81$
$= 160 + 106 = 266$

$$\text{proj}_{\text{range}(T)} v = \langle v, w_1 \rangle w_1 + \langle v, w_2 \rangle w_2$$

$$= \frac{1}{7}(8)\begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix} + \frac{1}{\cancel{266}_{133}}(\cancel{100}^{50})\begin{pmatrix} -5 \\ 4 \\ 12 \\ 9 \end{pmatrix}$$

$133 = 7 \cdot 19$

$19 \cdot 8 = 152$

$$= \frac{1}{133}\left[\begin{pmatrix} 152 \\ 304 \\ -152 \\ 152 \end{pmatrix} + \begin{pmatrix} -250 \\ 200 \\ 600 \\ 450 \end{pmatrix}\right] = \begin{pmatrix} -\frac{98}{133} \\ \frac{504}{133} \\ \frac{448}{133} \\ \frac{602}{133} \end{pmatrix} \neq v$$

$\boxed{\text{so } v \text{ not in range}(T)}$

$$\boxed{\text{dist} = \sqrt{\left(\frac{231}{133}\right)^2 + \left(\frac{105}{133}\right)^2 + \left(\frac{84}{133}\right)^2 + \left(\frac{63}{133}\right)^2}}$$

532
-448
84

$\leftarrow w^\perp = (1,3,4,5) - \text{proj}_{\text{range}(T)} v$
$= \left(\frac{231}{133}, \frac{105}{133}, \frac{84}{133}, \frac{63}{133}\right)$

16, 2019

Problem 5 (20 points)

For each system, find a general solution. [10 points each]

① $$x_1'(t) = -2x_1(t) + 3x_2(t)$$
$$x_2'(t) = -2x_2(t)$$

② $$x_1'(t) = -x_1(t) - x_2(t)$$
$$x_2'(t) = 2x_1(t) - 3x_2(t)$$

① $\vec{x}' = \begin{bmatrix} -2 & 3 \\ 0 & -2 \end{bmatrix} \vec{x}$

$\lambda = -2, -2$ (upper triangular)

$\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ η

$(A + 2I)\eta = v$ $\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\eta = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix}$

Take $\eta = \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix}$

$$\boxed{\vec{x} = C_1 e^{-2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2\left(te^{-2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{-2t}\begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix}\right)}$$

② $\vec{x}' = \begin{bmatrix} -1 & -1 \\ 2 & -3 \end{bmatrix} \vec{x}$ $\begin{vmatrix} -1-x & -1 \\ 2 & -3-x \end{vmatrix} = 3 + 4x + x^2 + 2$
$= x^2 + 4x + 5$

$\lambda = \frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i$

$\lambda = -2 + i$ $\begin{bmatrix} 1-i & -1 \\ 2 & -1-i \end{bmatrix}$ $v = \begin{pmatrix} 1 \\ 1-i \end{pmatrix}$

$e^{(-2+i)t}\begin{pmatrix} 1 \\ 1-i \end{pmatrix} = e^{-2t}(\cos t + i \sin t)\begin{pmatrix} 1 \\ 1-i \end{pmatrix}$

$= \begin{pmatrix} e^{-2t}\cos t \\ e^{-2t}\cos t + e^{-2t}\sin t \end{pmatrix} + i \begin{pmatrix} e^{-2t}\sin t \\ e^{-2t}\sin t - e^{-2t}\cos t \end{pmatrix}$

$$\boxed{\vec{x} = C_1 e^{-2t}\begin{pmatrix} \cos t \\ \cos t + \sin t \end{pmatrix} + C_2 e^{-2t}\begin{pmatrix} \sin t \\ \sin t - \cos t \end{pmatrix}}$$

$\frac{1}{2\pi}$

Math 54 Final Exam (...

Jeffrey Kuan

... 16 2019

Problem 6 (40 points)

Calculate the Fourier series expansion for the function

$$f(\theta) = \theta \qquad -\pi < \theta < \pi$$

Write your answer both as a sum of complex exponentials, and as a sum of sines and cosines. Use the expansion and Plancherel's theorem to find the value of the convergent infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$f(\theta) = \theta \qquad \hat{f}(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)e^{-in\theta}\,d\theta$$

$$\hat{f}(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\theta\,d\theta = 0$$

$$n \neq 0 \quad \hat{f}(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\theta e^{-in\theta}\,d\theta = \left[\frac{\theta e^{-in\theta}}{-in}\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi}\frac{e^{-in\theta}}{in}\,d\theta\right]\frac{1}{2\pi}$$

$$u = \theta \qquad dv = e^{-in\theta}\,d\theta$$

$$du = d\theta \qquad v = \frac{e^{-in\theta}}{-in}$$

$$= \left[\underbrace{-\frac{\theta e^{-in\theta}}{in}\Big|_{-\pi}^{\pi}}_{①} + \underbrace{\left(\frac{e^{-in\theta}}{-(in)^2}\right)\Big|_{-\pi}^{\pi}}_{②}\right]\frac{1}{2\pi}$$

① $-\frac{\pi e^{-in\pi}}{in} - \left(-\frac{-\pi e^{in\pi}}{in}\right) = \frac{-\pi}{in}\left(e^{in\pi} + e^{-in\pi}\right) = \frac{-\pi}{in}\left(\cos(n\pi) + \cancel{i\sin(n\pi)} + \cos(n\pi) - \cancel{i\sin(n\pi)}\right)$

$$= \frac{2\pi(-1)^{n+1}}{in}$$

② $\frac{e^{-in\pi}}{-(in)^2} - \frac{e^{in\pi}}{-(in)^2} = \frac{1}{(in)^2}\left(e^{in\pi} - e^{-in\pi}\right) = \frac{1}{(in)^2}\left(\cancel{\cos(n\pi)} + i\sin(n\pi) - \cancel{\cos(n\pi)} + i\sin(n\pi)\right) = 0.$

$$n \neq 0 \quad \hat{f}(n) = \frac{1}{2\pi}\left(\frac{2\pi(-1)^{n+1}}{in}\right) = \frac{(-1)^{n+1}}{in}$$

So $\boxed{f(\theta) \sim \sum_{n\neq 0,\ n\in\mathbb{Z}} \frac{(-1)^{n+1}}{in} e^{in\theta}}$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{in}e^{in\theta} + \sum_{n=1}^{\infty}\frac{(-1)^{-n+1}}{-in}e^{-in\theta}$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{in}\left(\cos(n\theta) + i\sin(n\theta)\right) + \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{in}\left(-\cos(n\theta) + i\sin(n\theta)\right)$$

$$= \boxed{\sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\sin(n\theta)}$$

Plancherel's Theorem: $\frac{1}{2\pi}\int_{-\pi}^{\pi}|f(\theta)|^2\,d\theta = \sum_{n\in\mathbb{Z}}|\hat{f}(n)|^2$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}|f(\theta)|^2\,d\theta = \frac{1}{2\pi}\int_{-\pi}^{\pi}\theta^2\,d\theta = \frac{\pi^2}{3}$$

$$\sum_{n\in\mathbb{Z}}|\hat{f}(n)|^2 = \sum_{n=1}^{\infty}|\hat{f}(n)|^2 + \sum_{n=1}^{\infty}|\hat{f}(-n)|^2 \quad (\hat{f}(0) = 0)$$

$$= \sum_{n=1}^{\infty}\left|\frac{(-1)^{n+1}}{in}\right|^2 + \sum_{n=1}^{\infty}\left|\frac{(-1)^{-n+1}}{-in}\right|^2 = \sum_{n=1}^{\infty}\frac{1}{n^2} + \sum_{n=1}^{\infty}\frac{1}{n^2} = 2\sum_{n=1}^{\infty}\frac{1}{n^2}$$

So $\frac{\pi^2}{3} = 2\sum_{n=1}^{\infty}\frac{1}{n^2}$, so $\boxed{\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}}$

Problem 7 (20 points)

Consider the following six second order differential equations.

$$y'' + 2y' + 17y = 0 \tag{1}$$

$$y'' - 3y' + 2y = 2x \tag{2}$$

$$y'' - 2y' + 17y = 0 \tag{3}$$

$$y'' + 2y' + y = 0 \tag{4}$$

$$y'' + 3y' + 2y = 0 \tag{5}$$

Match these differential equations to a graph of one of their particular solutions **on the next page**, which shows all of the figures[1]. Note that every equation here will be matched to a different graph, but not all graphs on the next page will be used.

(1) $r^2 + 2r + 17 = 0$

$r = \dfrac{-2 \pm \sqrt{-64}}{2} = -1 \pm 4i$

$C_1 e^{-x} \cos(4t) + C_2 e^{-x} \sin(4t)$

so (1) is graph $\boxed{D}$

(2) $r^2 - 3r + 2 = 0$

$(r-1)(r-2) = 0 \quad r = 1, 2$

particular soln is linear $+ C_1 e^{t} + C_2 e^{t}$

so (2) is graph $\boxed{B}$

(3) $r^2 - 2r + 17 = 0$

$r = 1 \pm 4i \quad C_1 e^{x} \cos(4t) + C_2 e^{x} \sin(4t)$

so (3) is graph $\boxed{F}$

(4) $r^2 + 2r + 1 = 0$

$(r+1)^2 = 0 \quad r = -1, -1 \quad C_1 e^{-t} + C_2 t e^{-t}$

te^{-t} looks like graph $\boxed{A}$

e^{-t} looks like graph $\boxed{E}$

so (4) is either $\boxed{A \text{ or } E}$

(either answer is acceptable)

END OF EXAM

(Figures on next page)

(5) $r^2 + 3r + 2 = 0$

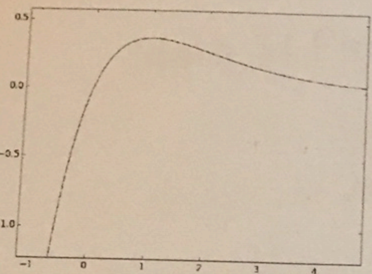
$(r+1)(r+2) = 0$

$r = -1, -2$

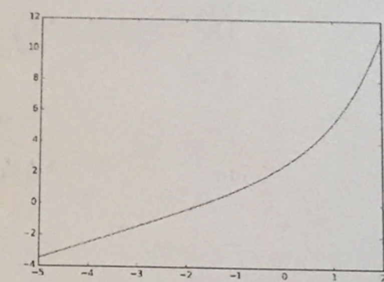
$C_1 e^{-t} + C_2 e^{-2t}$

(5) is $\boxed{E}$

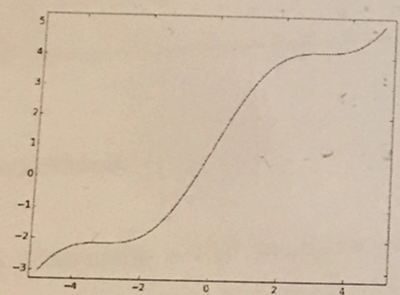
[1] Figures generated using Matplotlib



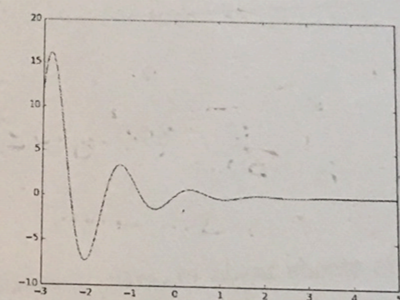
(a) Graph (A)



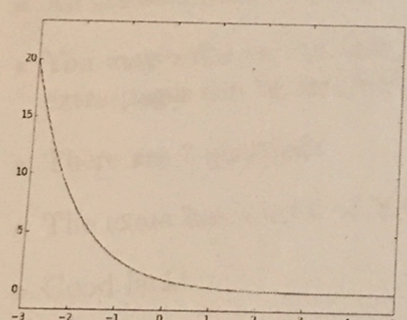
(b) Graph (B)



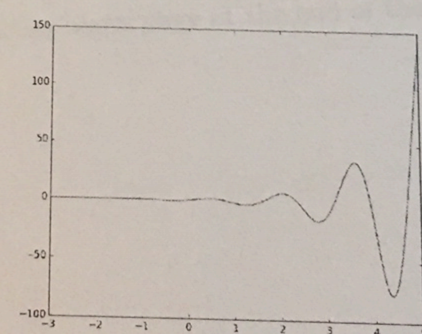
(c) Graph (C)



(d) Graph (D)



(e) Graph (E)



(f) Graph (F)