

$$1) \int \frac{(\ln x)^2}{\sqrt{x}} dx = 2\sqrt{x} (\ln x)^2 - 4 \int \frac{\ln x}{\sqrt{x}} dx$$

$$u = (\ln x)^2 \quad dv = \frac{1}{\sqrt{x}} dx \quad u = \ln x \quad dv = \frac{1}{\sqrt{x}} dx$$

$$du = \frac{2 \ln x}{x} \quad v = 2\sqrt{x} \quad du = \frac{1}{x} dx \quad v = 2\sqrt{x}$$

$$= 2\sqrt{x} (\ln x)^2 - 4 \left(2\sqrt{x} \ln x - \int \frac{2}{\sqrt{x}} dx \right)$$

$$= \boxed{2\sqrt{x} (\ln x)^2 - 8\sqrt{x} \ln x + 16\sqrt{x} + C}$$

$$\int \frac{1}{x \ln x \ln(\ln x)} dx = \int \frac{1}{u} du = \ln|u| = \boxed{\ln|\ln(\ln x)| + C}$$

$$\text{Let } u = \ln(\ln x).$$

$$du = \frac{1}{\ln x} \cdot \frac{1}{x} dx$$

$$\int e^{-2x} \cos(3x) dx = \frac{1}{3} e^{-2x} \sin(3x) + \frac{2}{3} \int e^{-2x} \sin(3x) dx$$

$$u = e^{-2x} \quad dv = \cos(3x) dx$$

$$du = -2e^{-2x} \quad v = \frac{1}{3} \sin(3x)$$

$$u = e^{-2x} \quad dv = \sin(3x) dx$$

$$du = -2e^{-2x} \quad v = -\frac{1}{3} \cos(3x)$$

$$= \frac{1}{3} e^{-2x} \sin(3x) + \frac{2}{3} \left(-\frac{1}{3} e^{-2x} \cos(3x) - \frac{2}{3} \int e^{-2x} \cos(3x) dx \right)$$

$$\int e^{-2x} \cos(3x) dx = \frac{1}{3} e^{-2x} \sin(3x) - \frac{2}{9} e^{-2x} \cos(3x) - \frac{4}{9} \int e^{-2x} \cos(3x) dx$$

$$\frac{13}{9} \int e^{-2x} \cos(3x) dx = \frac{1}{3} e^{-2x} \sin(3x) - \frac{2}{9} e^{-2x} \cos(3x)$$

$$\int e^{-2x} \cos(3x) dx = \boxed{\frac{9}{13} \left(\frac{1}{3} e^{-2x} \sin(3x) - \frac{2}{9} e^{-2x} \cos(3x) \right) + C}$$

$$\int \frac{x}{\sqrt{x^2-4}} dx = \int \frac{1}{\sqrt{u}} \frac{1}{2} du = \sqrt{u} + C = \boxed{\sqrt{x^2-4} + C}$$

$$u = x^2 - 4.$$

$$\frac{1}{2} du = x dx$$

$$\int \frac{x^4 - x}{x^3 - x} dx = \int \frac{x^3 - 1}{x^2 - 1} dx = \int x + \frac{x-1}{x^2-1} dx$$

$$x^2 + 0x - 1 \overline{) x^3 + 0x^2 + 0x - 1}$$

$$\begin{array}{r} x + \frac{x-1}{x^2-1} \\ -(x^3 + 0x^2 - x) \\ \hline x - 1 \end{array} = \int x + \frac{1}{x+1} dx = \boxed{\frac{1}{2}x^2 + \ln|x+1| + C}$$

$$\int \sin^2 x \cos^2 x dx = \int \left(\frac{1 - \cos(2x)}{2} \right) \left(\frac{1 + \cos(2x)}{2} \right) dx$$

$$\text{Use } \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$= \frac{1}{4} \int 1 - \cos^2(2x) dx$$

$$= \frac{1}{4} \int 1 - \frac{1 + \cos(4x)}{2} dx$$

$$= \frac{1}{4} \int \frac{1}{2} - \frac{1}{2} \cos(4x) dx$$

$$= \frac{1}{8} \int 1 - \cos(4x) dx$$

$$= \boxed{\frac{1}{8}x - \frac{1}{32} \sin(4x) + C}$$

$$\int \frac{x^2+2x}{(x^2+4x+5)^2} dx$$

x^2+4x+5 is irreducible ($b^2-4ac=4^2-4(1)(5)<0$)

$$\frac{x^2+2x}{(x^2+4x+5)^2} = \frac{Ax+B}{x^2+4x+5} + \frac{Cx+D}{(x^2+4x+5)^2}$$

$$x^2+2x = (x^2+4x+5)(Ax+B) + Cx+D$$

$$x^2+2x = Ax^3 + 4Ax^2 + 5Ax + Bx^2 + 4Bx + 5B + Cx + D$$

$$= Ax^3 + (4A+B)x^2 + (5A+4B+C)x + (5B+D)$$

$$A=0 \quad 4A+B=1 \quad 5A+4B+C=2 \quad 5B+D=0$$

$$B=1 \quad 4+C=2 \quad D=-5$$

$$C=-2$$

$$\int \frac{0x+1}{x^2+4x+5} dx + \int \frac{-2x-5}{(x^2+4x+5)^2} dx$$

$$\int \frac{1}{(x+2)^2+1} dx$$

$$= \arctan(x+2) + C$$

$$\int \frac{-2x-5}{((x+2)^2+1)^2} dx$$

$$= \int \frac{-2(u-2)-5}{(u^2+1)^2} du$$

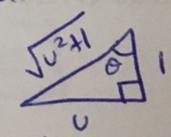
$$= \int \frac{-2u-1}{(u^2+1)^2} du$$

Let $u=x+2$
 $x=u-2$
 $dx=du$

$$= \int \frac{-2u}{(u^2+1)^2} du - \int \frac{1}{(u^2+1)^2} du$$

Use $v=u^2+1$
 $2v dv = du$

$$\frac{1}{u^2+1} + C$$



$$\int \frac{1}{\sec^4 \theta} \sec^2 \theta d\theta \quad \tan \theta = u \quad \sqrt{u^2+1} = \sec \theta$$

$$\sec^2 \theta d\theta = du$$

$$= \int \cos^2 \theta d\theta = \int \frac{1+\cos(2\theta)}{2} d\theta = \frac{\theta}{2} - \frac{1}{4} \sin(2\theta) + C$$

$$= \frac{\theta}{2} - \frac{1}{2} \sin \theta \cos \theta$$

$$= \frac{1}{2} \arctan u - \frac{1}{2} \left(\frac{u}{\sqrt{u^2+1}} \right) \left(\frac{1}{\sqrt{u^2+1}} \right)$$

$$= \frac{1}{2} \arctan u - \frac{u}{2(u^2+1)}$$

ANS:

$$\arctan(x+2) + \frac{1}{(x+2)^2+1}$$

$$+ \frac{1}{2} \arctan(x+2) - \frac{x+2}{2((x+2)^2+1)}$$

$$\int \frac{x^2}{(x+1)(x^2-4x+4)} dx = \int \frac{x^2}{(x+1)(x-2)^2} dx$$

$$\frac{x^2}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2 = (x-2)^2 A + (x+1)(x-2)B + (x+1)C$$

$$x=2: \quad 4 = 3C \quad C = \frac{4}{3}$$

$$x=-1: \quad 1 = 9A \quad A = \frac{1}{9}$$

$$x^2 = (x-2)^2 \frac{1}{9} + (x+1)(x-2)B + (x+1) \frac{4}{3}$$

$$x=0: \quad 0 = \frac{4}{9} - 2B + \frac{4}{3}$$

$$2B = \frac{16}{9} \quad B = \frac{8}{9}$$

$$= \int \frac{1}{9} \frac{1}{x+1} dx + \int \frac{8}{9} \frac{1}{x-2} dx + \int \frac{4}{3} \frac{1}{(x-2)^2} dx$$

$$= \boxed{\frac{1}{9} \ln|x+1| + \frac{8}{9} \ln|x-2| - \frac{4}{3} \frac{1}{x-2} + C}$$

$$\int \frac{\sqrt{x}}{x+1} dx$$

$$\text{Let } u = \sqrt{x}.$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$= \int \frac{1}{2\sqrt{x}} \frac{2x}{x+1} dx = \int \frac{2u^2}{u^2+1} du$$

$$= \int 2 - \frac{2}{u^2+1} du$$

$$= 2u - 2 \arctan u + C$$

$$= \boxed{2\sqrt{x} - 2 \arctan(\sqrt{x}) + C}$$

$$\int \frac{\tan^3 x}{\sec x} dx = \int \frac{\tan^2 x}{\sec x} \tan x dx = \int \frac{\sec^2 x - 1}{\sec x} \tan x dx$$

$$= \int \frac{\sec^2 x - 1}{\sec^2 x} \sec x \tan x dx = \int \frac{u^2 - 1}{u^2} du = \int 1 - \frac{1}{u^2} du$$

$$u = \sec x$$

$$du = \sec x \tan x dx$$

$$= u + \frac{1}{u} + C$$

$$= \boxed{\sec x + \cos x + C}$$

$$2) y = e^x$$

$$y' = e^x$$

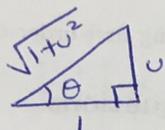
$$2\pi \int_{-1}^1 e^x \sqrt{1+e^{2x}} dx$$

$$2\pi \int_{-1}^1 y \sqrt{1+(f'(x))^2} dx$$

$$\text{Let } u = e^x. \quad du = e^x dx$$

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$$\int e^x \sqrt{1+e^{2x}} dx = \int \sqrt{1+u^2} du$$



$$\tan \theta = u$$

$$\sec \theta = \sqrt{1+u^2}$$

$$\sec^2 \theta d\theta = du$$

$$= \int \sec \theta \sec^2 \theta d\theta$$

$$= \int \sec^3 \theta d\theta$$

$$= \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| + C$$

$$= \boxed{\frac{1}{2} u \sqrt{1+u^2} + \frac{1}{2} \ln |u + \sqrt{1+u^2}| + C}$$

$$2\pi \int_{-1}^1 e^x \sqrt{1+e^{2x}} dx$$

$$= 2\pi \left(\frac{1}{2} e^x \sqrt{1+e^{2x}} + \frac{1}{2} \ln |e^x + \sqrt{1+e^{2x}}| \right) \Big|_{-1}^1$$

$$= \boxed{2\pi \left(\frac{1}{2} e \sqrt{1+e} + \frac{1}{2} \ln |e + \sqrt{1+e}| - \frac{1}{2} e^{-1} \sqrt{1+e^{-2}} - \frac{1}{2} \ln |e^{-1} + \sqrt{1+e^{-2}}| \right)}$$

$$3) f(x) = \frac{1}{3} (1+4x)^{\frac{3}{2}} \quad f'(x) = \frac{1}{2} (1+4x)^{\frac{1}{2}} \cdot 4 = 2(1+4x)^{\frac{1}{2}}$$

$$L = \int_{+1}^3 \sqrt{1+(2(1+4x)^{\frac{1}{2}})^2} dx = \int_{-1}^3 \sqrt{1+4(1+4x)} dx$$

$$= \int_{-1}^3 \sqrt{5+16x} dx = \frac{1}{16} \left(\frac{2}{3} \right) (5+16x)^{\frac{3}{2}} \Big|_{+1}^3$$

$$= \boxed{\frac{1}{24} (53^{\frac{3}{2}} - 21^{\frac{3}{2}})}$$

$$4) f(x) = \frac{1}{16}x^4 + \frac{1}{2x^2}$$

$$f'(x) = \frac{1}{4}x^3 - \frac{1}{x^3}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{1}{4}x^3 - \frac{1}{x^3}\right)^2} dx$$

$$= \int_1^2 \sqrt{1 + \frac{1}{16}x^6 - \frac{1}{2} + \frac{1}{x^6}} dx = \int_1^2 \sqrt{\frac{1}{16}x^6 + \frac{1}{2} + \frac{1}{x^6}} dx$$

$$= \int_1^2 \sqrt{\left(\frac{1}{4}x^3 + \frac{1}{x^3}\right)^2} dx$$

$$= \int_1^2 \left| \frac{1}{4}x^3 + \frac{1}{x^3} \right| dx = \int_1^2 \frac{1}{4}x^3 + \frac{1}{x^3} dx$$

$$= \frac{1}{16}x^4 - \frac{1}{2x^2} \Big|_1^2 = 1 - \frac{1}{8} - \left(\frac{1}{16} - \frac{1}{2}\right) \text{ since } \frac{1}{4}x^3 + \frac{1}{x^3} > 0 \text{ on } [1, 2].$$

$$= \frac{14}{16} - \left(-\frac{7}{16}\right) = \boxed{\frac{21}{16}}$$

$$SA = \int_1^2 2\pi \left(\frac{1}{16}x^4 + \frac{1}{2x^2}\right) \sqrt{1 + \left(\frac{1}{4}x^3 - \frac{1}{x^3}\right)^2} dx$$

$$= \int_1^2 2\pi \left(\frac{1}{16}x^4 + \frac{1}{2x^2}\right) \left| \frac{1}{4}x^3 + \frac{1}{x^3} \right| dx \leftarrow \text{same calculation as above.}$$

$$= \int_1^2 2\pi \left(\frac{1}{16}x^4 + \frac{1}{2x^2}\right) \left(\frac{1}{4}x^3 + \frac{1}{x^3}\right) dx \leftarrow$$

$$= \int_1^2 2\pi \left(\frac{1}{64}x^7 + \frac{1}{16}x + \frac{1}{8}x + \frac{1}{2x^5}\right) dx$$

$$= 2\pi \left(\frac{1}{512}x^8 + \frac{3}{32}x^2 + \left(-\frac{1}{8x^4}\right)\right) \Big|_1^2$$

$$= \boxed{2\pi \left(\frac{1}{512}2^8 + \frac{3}{32} \cdot 4 - \frac{1}{8 \cdot 16} - \frac{1}{512} - \frac{3}{32} + \frac{1}{8}\right)}$$

$$5) \sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n^3}$$

Consider $\sum |a_n| = \sum \frac{\ln n}{n^3}$.

Use Integral Test.

$$\int_1^{\infty} \frac{\ln x}{x^3} dx = \lim_{N \rightarrow \infty} \int_1^N \frac{\ln x}{x^3} dx$$

ASIDE: $\int \frac{\ln x}{x^3} dx$

$u = \ln x \quad dv = \frac{1}{x^3} dx = -\frac{1}{2x^2} \ln x + \int \frac{1}{2x^3} dx$

$du = \frac{1}{x} dx \quad v = -\frac{1}{2x^2}$

$= -\frac{1}{2x^2} \ln x - \frac{1}{4x^2} + C$

$= \lim_{N \rightarrow \infty} -\frac{\ln N}{2N^2} - \frac{1}{4N^2} + \frac{1}{4}$

$= \frac{1}{4} + \lim_{N \rightarrow \infty} -\frac{\ln N}{2N^2} - \frac{1}{4N^2}$

L'H $\downarrow = \frac{1}{4} + \lim_{N \rightarrow \infty} \frac{-\frac{1}{N}}{\frac{4}{N^2}}$

$= \frac{1}{4} + \lim_{N \rightarrow \infty} -\frac{1}{4N^2} = \frac{1}{4}$

So $\sum |a_n| = \sum \frac{\ln n}{n^3}$ converges by Integral Test,

so $\sum a_n = \sum (-1)^n \frac{\ln n}{n^3}$ converges absolutely.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{e^n}$$

Consider $\sum |a_n| = \sum n^2 e^{-n}$.

Use Integral Test. $\int_1^{\infty} x^2 e^{-x} dx = \lim_{N \rightarrow \infty} \int_1^N x^2 e^{-x} dx$

ASIDE: $\int x^2 e^{-x} dx = -x^2 e^{-x} + \int 2x e^{-x} dx$

$= \lim_{N \rightarrow \infty} -N^2 e^{-N} - 2N e^{-N} - 2e^{-N} + e^{-1} + 2e^{-1} + 2e^{-1}$

$u = x^2 \quad dv = e^{-x} dx$

$du = 2x \quad v = -e^{-x}$

$= -x^2 e^{-x} - 2x e^{-x} + \int 2e^{-x} dx$

$u = 2x \quad dv = e^{-x} dx = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C$

$du = 2 dx \quad v = -e^{-x}$

$= 5e^{-1} + \lim_{N \rightarrow \infty} -\frac{N^2}{e^N} + \lim_{N \rightarrow \infty} -\frac{2N}{e^N}$

L'H $\downarrow = 5e^{-1} + \lim_{N \rightarrow \infty} \frac{-2N}{e^N} + \lim_{N \rightarrow \infty} \frac{-2}{e^N} \rightarrow 0$

L'H $\downarrow = 5e^{-1} + \lim_{N \rightarrow \infty} \frac{-2}{e^N} \rightarrow 0 = 5e^{-1}$

So $\sum |a_n| = \sum \frac{n^2}{e^n}$ converges by Integral Test,

so $\sum a_n = \sum (-1)^n \frac{n^2}{e^n}$ converges absolutely.