Math 1A Final (Optional) Practice 3

November 26, 2018

This is optional practice for the final. We will have an optional practice every week. If you hand it in, I will check it for you and give you feedback, but it is optional, as stated before. If you would like personalized feedback on your work on this worksheet, please hand this in by **December 3, 2018**.

Question 1

Compute the following limits.

$$\lim_{x\to\infty} \frac{\arctan(\sin(e^x))}{\sqrt{x}+\arctan(x)+1}$$

$$\lim_{x\to 0} \tan\left(x^2 \sin\left(\frac{1}{x}\right)+\frac{5\pi}{6}\right)$$

$$\lim_{x\to e^+} \frac{x^2-2x}{(1-\ln(x))^7}$$

Question 2

Calculate the derivatives of the following functions.

$$f(x)=\sec(\ln|\sin(x)|)$$

$$f(x)=\arccos\left(\sqrt{x-\sqrt{x}}\right)$$

$$f(x)=\left(\sqrt{x}\right)^{x^2+1}$$

$$f(x)=3\cdot 2^{\sin(2^x)}$$

$$f(x)=\frac{x^2}{(2+\sin(x))^x}$$

Question 3

Find the maximum area of a triangle inscribed in the ellipse $x^2 + \frac{1}{9}y^2 = 1$ that has vertices at $(0, 3)$, (x, y), and $(-x, y)$, where (x, y) is on the ellipse. (Note that x and y could potentially be negative also).

Next, find the maximum area of a triangle inscribed in the ellipse $x^2 + \frac{1}{9}y^2 = 1$ that has vertices at $(1, 0)$, (x, y), and $(x, -y)$, where (x, y) is on the ellipse.

Question 4

Find the equation of the tangent and normal line to the graph of

$$x^2 \arcsin(y) + e^{\sqrt{xy}} + \sqrt{1 + \sqrt{xy}} = 2 - 3x$$

at the point $(0, 1)$.

Question 5

Consider the function $f(x) = \arcsin(x) + \tan\left(\frac{\pi}{2}x\right) + 3e^x$. Find the domain of this function. Then prove, using the Intermediate Value Theorem and the Mean Value Theorem that f has exactly one zero on its domain. (Hint: Consider $\lim_{x \to -1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$).

Question 6

Sketch the graph of

$$f(x) = \frac{2x^3 - x^2 - 3x}{x^2 - x}$$

Question 7

Sketch the graph of

$$f(x) = x \cdot \arctan(x) - \int_0^x \arctan(t)dt$$