Math 1A Final (Optional) Practice 2

November 19, 2018

This is optional practice for the final. We will have an optional practice every week. If you hand it in, I will check it for you and give you feedback, but it is optional, as stated before. If you would like personalized feedback on your work on this worksheet, please hand this in by **November 26, 2018**.

Question 1: Gnarly Trigs

Compute the following quantities.

$$\arcsin(1)$$

$$\arccos\left(-\frac{1}{\sqrt{2}}\right)$$

$$\arctan(\sqrt{3})$$

$$\arcsin\left(\sin\left(\frac{14\pi}{5}\right)\right)$$

(Hint for the previous one: You cannot compute $\sin\left(\frac{14\pi}{5}\right)$ directly, but maybe you can somehow try to reason your way through this one.)

$$\sin\left(\arcsin(0.7)\right)$$

$$\cos\left(\operatorname{arcsec}\left(\frac{5}{3}\right)\right)$$

$$\csc\left(\arctan(x)\right)$$

Question 2: Limit Warmup

Compute the following limits. These are a warmup for the difficult limits in the next question.

$$\lim_{x\to 2}\left(\frac{x^2-4}{x^3-8}\right)$$

(Remark: Do this both ways we have learned, factoring and L'Hopital's. Check that you get the same answer.)

$$\lim_{x\to \frac{\pi}{2}^+}\left(\frac{-x^3}{1-\sin^4(x)}\right)$$

(Remark: Can't do L'Hopital's rule here. Why?)

$$\lim_{x\to\infty}\left(1+\frac{1}{4\sqrt{x}}\right)^{\sqrt{x+1}}$$

$$\lim_{x\to 0^+}\left(\ln(x)-\frac{1}{x}\right)$$

(Hint: Factor.)

Question 3: Hard Limits

Compute the following limits.

$$\lim_{x\to-\infty}\frac{\sin(x)\cos(x)}{x^2e^{x^4}+|x|}$$

(Hint: Squeeze. Note that the numerator is between -1 and 1. Why?)

$$\lim_{x\to\infty}\frac{\sqrt{x^3+2}}{2x^{3/2}+\sin(x)}$$

(Hint: Divide by highest order term, then squeeze.)

$$\lim_{h\to 0}\frac{e^{x+h}-2e^x+e^{x-h}}{h^2}$$

(Hint: If you decide to do L'Hopital's rule, be extremely careful about what variable you are taking the derivative with respect to. Hint 2: The variable you should be differentiating with respect to is not x. Why?)

$$\lim_{x\to\infty}\frac{3e^{-x}+1}{\arctan(2x)-\arctan(x)}$$

(Hint: You can't use L'Hopital's rule here! Why? Draw a graph of arctan and see if you can figure out what is going on with the denominator.)

$$\lim_{x\to 0}\frac{\int_0^{x^2}\cos(t^{1/3})dt}{4x^2}$$

(Hint: You cannot actually compute the integral in the numerator, so don't try that. But can we use L'Hopital somehow?)

Question 4: Transformations

Pay very close attention! Here is a common mistake people make. As an example, a horizontal shift to the left by 2 causes $f(x)$ to become $f(x+2)$. So if $\tan(2x-3)$ is shifted to the left by 3, the result would not be $\tan(2x)$. It would be $\tan(2(x+3)-3)=\tan(2x+3)$! Keeping this in mind, consider the following question.

What transformations are needed to change the graph of $y=-\frac{1}{3}\tan(-2x+3)+2$ into the graph of $y=\tan(x+2)-1$?

Answer this question. Then consider the two answers below. Are they correct?

- First answer: Shift down by 2, vertical squeeze by 3, reflect across x-axis, shift left by $3/2$, horizontal stretch by 2, reflect across y-axis, shift down by 3, shift left by 2.
- Second answer: Reflect across x-axis, shift down by 2, vertical squeeze by 3, reflect across x axis, horizontal stretch by 2, shift left by 3, shift down by 1, shift left by 2.

The last two questions are variations of the Midterm 2 questions on curve sketching and optimization.

Question 5: Curve Sketching

Sketch the curve

$$f(x) = \frac{\ln|x|}{x}$$

(Hint: The derivative of $\ln|x|$ is $\frac{1}{x}$. Or a better way to do it is to use the fact that $|x| = x$ if $x > 0$ and $|x| = -x$ if $x < 0$, and split this into two problems. Or an even better way is to notice that f is an odd function, so you only really need to graph it for $x \geq 0$.)

Question 6: Optimization

A rectangular prism with a square base is inscribed in a sphere of radius 2. Find the maximum possible volume of such a rectangular prism. (Hint: Draw a really good picture and use the Pythagorean theorem more than once.)