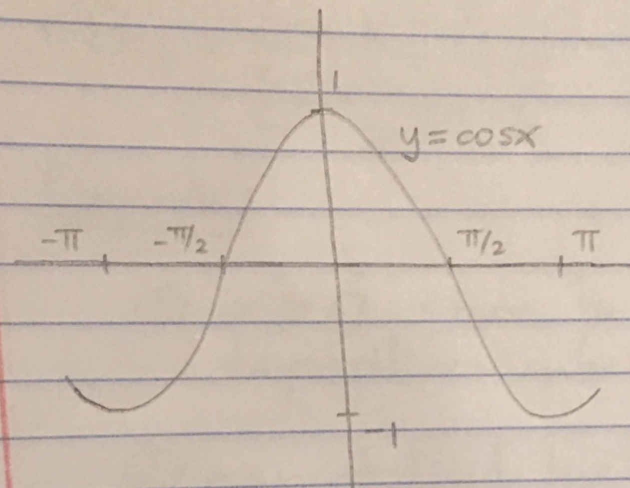
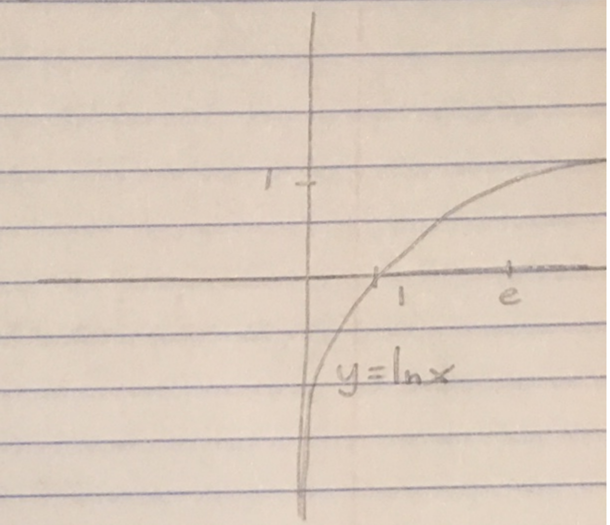
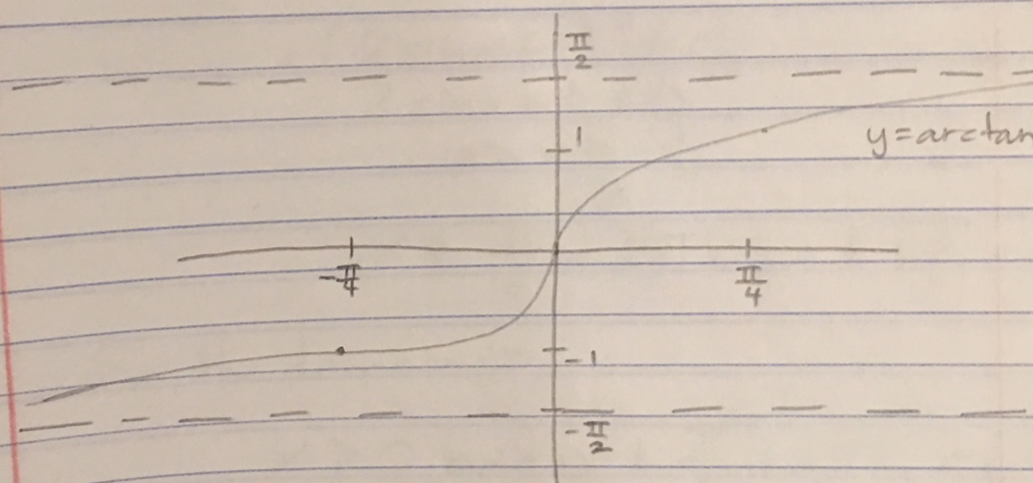
Final Practice 1 Answers

1)







2) $$\ln\left(\frac{x^2y^3}{e^3\sqrt[3]{z}}\right) = \ln(x^2)+\ln(y^3)-\ln(e^3)-\ln(\sqrt[3]{z})$$

$$=\ln(x^2)+\ln(y^3)-\ln(e^3)-\ln(z^{\frac{1}{3}})$$

$$=2\ln x+3\ln y-3\underbrace{\ln e}_{\ln e=1}-\frac{1}{3}\ln z$$

$$=2\ln x+3\ln y-\frac{1}{3}\ln z-3$$

3) $f(x) = \dfrac{\overbrace{3\ln x + 2}^{①}}{\underbrace{(2\sin x + 1)^2}_{②}}$

Domain:

① $x > 0$ since ln can only be applied to positive numbers

② cannot divide by 0, so

$$(2\sin x + 1)^2 \neq 0$$
$$2\sin x + 1 \neq 0$$
$$\sin x \neq -\frac{1}{2}$$

$$x \neq \frac{\pi}{6} + 2\pi k,\ x \neq \frac{5\pi}{6} + 2\pi k$$

So the domain is

$x > 0$ and $x \neq \frac{\pi}{6} + 2\pi k$, $x \neq \frac{5\pi}{6} + 2\pi k$ for integers k

Zeros: $3\ln x + 2 = 0$

$$\ln x = -\frac{2}{3}$$

$$\boxed{x = e^{-\frac{2}{3}}}$$

$$f'(x) = \frac{(2\sin x + 1)^2\left(\frac{3}{x}\right) - (3\ln x + 2)(2\cdot(2\sin x + 1)(2\cos x))}{(2\sin x + 1)^4}$$

(Quotient rule)

4) The domain of $f(x) = \arcsin x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

FALSE The domain of $f(x) = \arcsin x$ is $[-1, 1]$. The <u>range</u> of $f(x) = \arcsin x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

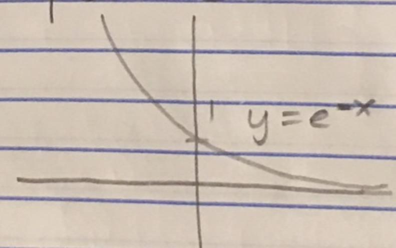
$\ln(x+y) = \ln x \ln y$

FALSE The property should be

* $\ln(xy) = \ln x + \ln y$!

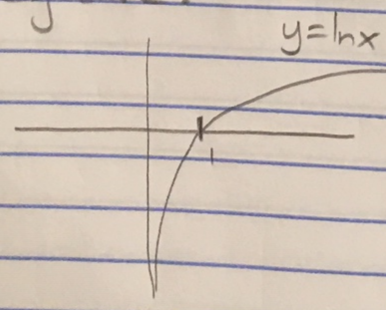
e^{-x} is always positive.

TRUE



$\ln(x)$ is never negative.

FALSE



e.g. $\ln(\frac{1}{e}) = -1$

$$\ln\left(\frac{x}{y}\right) = \frac{\ln x}{\ln y}$$

FALSE The property is

$$* \quad \ln\left(\frac{x}{y}\right) = \ln x - \ln y .$$

$\arctan(\tan\theta) = \theta$ for every angle θ.

FALSE

- not true for $\theta = \frac{\pi}{2} + \pi k$ since $\tan\theta$ is undefined.

- A better answer: Because of the range of $\arctan x$, this is not true for any θ not in $(-\frac{\pi}{2}, \frac{\pi}{2})$. For example,

$$\arctan\left(\tan\frac{5\pi}{4}\right) = \frac{\pi}{4} .$$