Math 1A Final Optional HW 1

1) The domain of $f(x) = \sin x$ is $\mathbb{R}$ and its range is $[-1, 1]$.

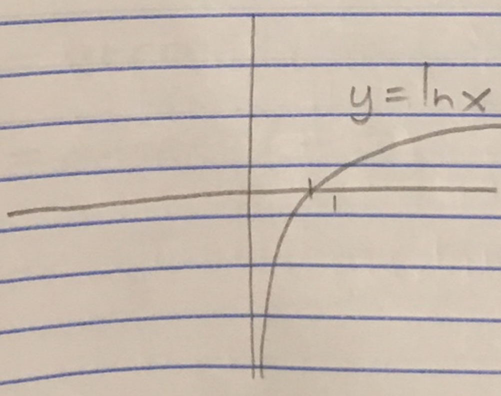
The domain of $f(x) = \arcsin x$ is $[-1, 1]$ and its range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

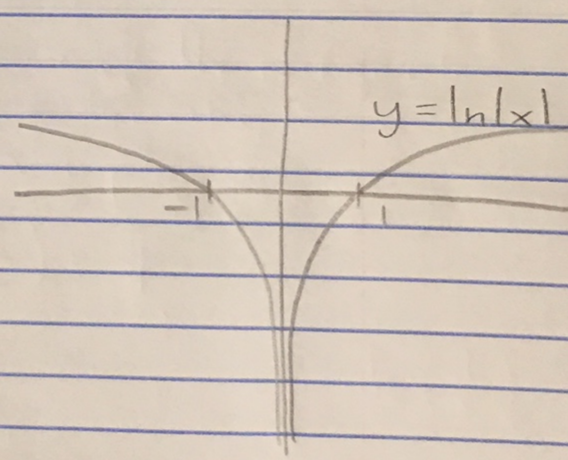
The domain of $f(x) = \arccos x$ is $[-1, 1]$ and its range is $[0, \pi]$.

The domain of $f(x) = \arctan x$ is $\mathbb{R}$ and its range is $(-\frac{\pi}{2}, \frac{\pi}{2})$.

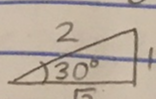
The domain of $f(x) = \ln x$ is $(0, \infty)$ and its range is $\mathbb{R}$.

2)

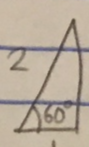




3) $\sin\left(\frac{11\pi}{6}\right) = \frac{1}{2}$



$$\cot\left(-\frac{4\pi}{3}\right) = \frac{1}{\tan(-\frac{4\pi}{3})} = \frac{1}{\tan(\frac{2\pi}{3})}$$



$$= \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$\log_4(2) = \frac{1}{2}$

$4^x = 2$

$(2^2)^x = 2^1$

$2^{2x} = 2^1 \Rightarrow 2x = 1 \quad x = \frac{1}{2}$

$\log_{27}\left(\frac{1}{\sqrt{3}}\right) = -\frac{1}{6}$

$27^x = \frac{1}{\sqrt{3}}$

$(3^3)^x = 3^{-\frac{1}{2}}$

$3^{3x} = 3^{-\frac{1}{2}} \Rightarrow 3x = -\frac{1}{2} \quad x = -\frac{1}{6}$

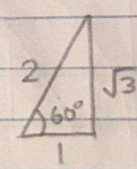
$\arccos\left(\cos\left(\frac{20\pi}{3}\right)\right)$

$= \arccos\left(\cos\left(\frac{2\pi}{3}\right)\right)$ since $\frac{2\pi}{3} + 6\pi = \frac{20\pi}{3}$ and $\cos(x)$ has period 2π

$= \arccos\left(-\frac{1}{2}\right)$

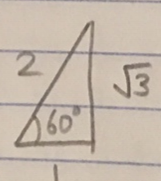
$= \frac{2\pi}{3}$

What angle θ between 0 and π has $\cos\theta = -\frac{1}{2}$?



4) $\sin(3x) = \frac{\sqrt{3}}{2}$

Let $y = 3x$.



$\sin y = \frac{\sqrt{3}}{2}$

$y = \frac{\pi}{3} + 2\pi k, \ \frac{2\pi}{3} + 2\pi k$

$3x = \frac{\pi}{3} + 2\pi k, \ \frac{2\pi}{3} + 2\pi k \Rightarrow \boxed{x = \frac{\pi}{9} + \frac{2\pi}{3}k, \ \frac{2\pi}{9} + \frac{2\pi}{3}k}$

5) $$f(x) = \frac{\overbrace{\ln(x^2-4x+3)}^{③}}{\underbrace{\underbrace{\tan(2x)}_{①}+1}_{②}}$$

Domain:

① tan cannot take in any values of the form $\frac{\pi}{2} + \pi k$.

So $2x \neq \frac{\pi}{2} + \pi k$

$x \neq \frac{\pi}{4} + \frac{\pi}{2}k$

② cannot divide by zero

$\tan(2x) + 1 \neq 0$

$\tan(2x) \neq -1$

$2x \neq \frac{3\pi}{4} + \pi k$ (tan has period π)

$x \neq \frac{3\pi}{8} + \frac{\pi}{2}k$

③ can only take ln of positive numbers

$x^2 - 4x + 3 > 0$

$(x-1)(x-3) > 0$ $\qquad x < 1$ or $x > 3$

+	1	−	3	+

So domain of f is: $\boxed{\begin{array}{c} x < 1 \text{ or } x > 3, \\ x \neq \frac{\pi}{4} + \frac{\pi}{2}k, \frac{3\pi}{8} + \frac{\pi}{2}k \\ \text{for integers } k \end{array}}$

Zeros: $\ln(x^2-4x+3)=0$

$$x^2-4x+3=1$$

$$x^2-4x+2=0$$

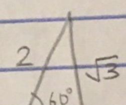
$$x=\frac{4\pm\sqrt{16-8}}{2}$$

$$=\boxed{2\pm\sqrt{2}}$$

6) 0.5

$\arctan(10)$

$-\frac{\pi}{3}=\arctan(-\sqrt{3})$



What angle θ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ has $\sin\theta=-1$?

$\arcsin(-1)=-\frac{\pi}{2}$

$\frac{\pi}{2}$

$\arctan(1)=\frac{\pi}{4}\simeq 0.75$

$\arctan(-10)$

Use two facts about $\arctan x$.

- It is strictly increasing: if $x>y$, then $\arctan(x)>\arctan(y)$.
- For any real number x,

$$-\frac{\pi}{2}<\arctan x<\frac{\pi}{2}$$

So by Fact 2 and the fact that

$$-\frac{\pi}{2}<\frac{1}{2}=0.5<\frac{\pi}{2},$$

we have $\arcsin(-1)=-\frac{\pi}{2}$ is the smallest value and $\frac{\pi}{2}$ is the largest value.

Then, order the negative values.

$$-\frac{\pi}{2} = \arcsin(-1), \quad \arctan(-10),$$

$$-\frac{\pi}{3} = \arctan(-\sqrt{3}) \quad \text{(smallest to largest)}$$

$-\frac{\pi}{2}$ is the smallest and since $-10 < -\sqrt{3}$,
we have that

$$\arctan(-10) < \arctan(-\sqrt{3}) = \frac{\pi}{3}.$$

Order the positive values.
(smallest to largest)

$$0.5, \quad \arctan(1) = \frac{\pi}{4}, \quad \arctan(10), \quad \frac{\pi}{2}$$

$0.5 < \arctan(1) = \frac{\pi}{4}$ since $\frac{\pi}{4} > \frac{3}{4} = 0.75$.

$\arctan(1) < \arctan(10)$ since $1 < 10$.

So the final order is

$$-\frac{\pi}{2} = \arcsin(-1)$$
$$\arctan(-10)$$
$$-\frac{\pi}{3} = \arctan(-\sqrt{3})$$
$$0.5$$
$$\frac{\pi}{4} = \arctan(1)$$
$$\arctan(10)$$
$$\frac{\pi}{2}$$