Discussion 11/9/18

Problem Set 1

1) · Antiderivative for $f(x) = x^2 + 3x + 2$ is

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + C$$

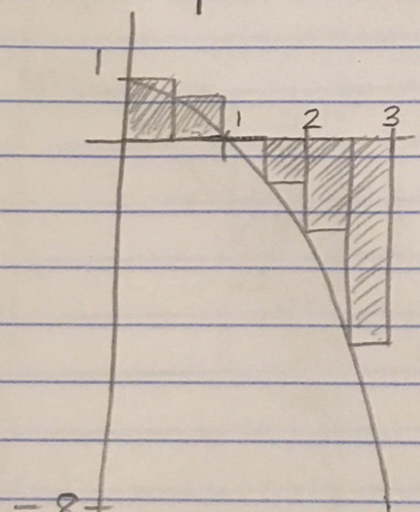
· Antiderivative for $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + e^x$ is

$$\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + e^x + C$$

· Antiderivative for $f(x) = \sin(2x) + \cos\left(\frac{x}{2}\right) + \sec^2 x$ is

$$-\frac{1}{2}\cos(2x) + 2\sin\left(\frac{x}{2}\right) + \tan x + C$$

2) · Left endpoint Riemann sum



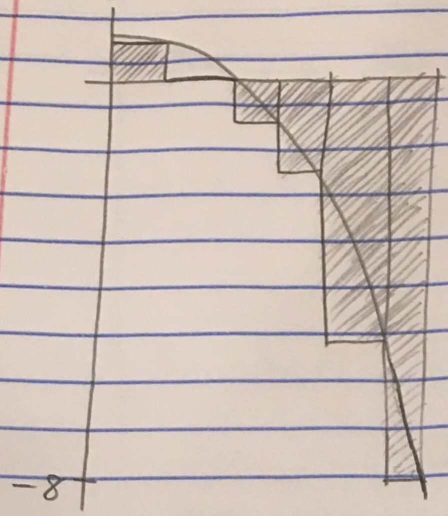
$$\int_0^3 1 - x^2 \, dx$$

$$\approx \frac{1}{2}f(0) + \frac{1}{2}f\left(\frac{1}{2}\right) + \frac{1}{2}f(1) + \frac{1}{2}f\left(\frac{3}{2}\right) + \frac{1}{2}f(2) + \frac{1}{2}f\left(\frac{5}{2}\right)$$

$$= \frac{1}{2}(1) + \frac{1}{2}\left(\frac{3}{4}\right) + \frac{1}{2}(0) + \frac{1}{2}\left(-\frac{5}{4}\right) + \frac{1}{2}(-3) + \frac{1}{2}\left(-\frac{21}{4}\right)$$

$$= \boxed{-3.875}$$

· Rightendpoint Riemann sum



$$\int_0^3 1-x^2\,dx$$

$$\approx \frac{1}{2}f\left(\frac{1}{2}\right)+\frac{1}{2}f(1)+\frac{1}{2}f\left(\frac{3}{2}\right)$$

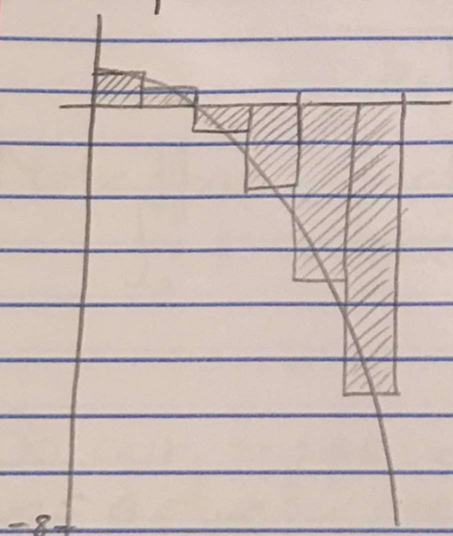
$$+\frac{1}{2}f(2)+\frac{1}{2}f\left(\frac{5}{2}\right)+\frac{1}{2}f(3)$$

$$=\frac{1}{2}\left(\frac{3}{4}\right)+\frac{1}{2}(0)+\frac{1}{2}\left(-\frac{5}{4}\right)$$

$$+\frac{1}{2}(-3)+\frac{1}{2}\left(-\frac{21}{4}\right)+\frac{1}{2}(-8)$$

$$\approx \boxed{-8.375}$$

· Midpoint rule Riemann sum



$$\int_0^3 1-x^2\,dx$$

$$\approx \frac{1}{2}f\left(\frac{1}{4}\right)+\frac{1}{2}f\left(\frac{3}{4}\right)+\frac{1}{2}f\left(\frac{5}{4}\right)$$

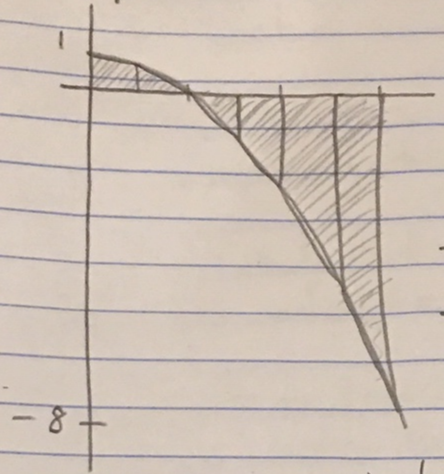
$$+\frac{1}{2}f\left(\frac{7}{4}\right)+\frac{1}{2}f\left(\frac{9}{4}\right)+\frac{1}{2}f\left(\frac{11}{4}\right)$$

$$=\frac{1}{2}\left(\frac{15}{16}\right)+\frac{1}{2}\left(\frac{7}{16}\right)+\frac{1}{2}\left(-\frac{9}{16}\right)$$

$$+\frac{1}{2}\left(-\frac{33}{16}\right)+\frac{1}{2}\left(-\frac{65}{16}\right)+\frac{1}{2}\left(-\frac{105}{16}\right)$$

$$\approx \boxed{-5.9375}$$

· Trapezoidal rule



$$\int_0^3 1-x^2\,dx$$

$$\approx \tfrac{1}{2}\left(f(0)+f\left(\tfrac{1}{2}\right)\right)\cdot\tfrac{1}{2} + \tfrac{1}{2}\left(f\left(\tfrac{1}{2}\right)+f(1)\right)\cdot\tfrac{1}{2} + \tfrac{1}{2}\left(f(1)+f\left(\tfrac{3}{2}\right)\right)\cdot\tfrac{1}{2} + \tfrac{1}{2}\left(f\left(\tfrac{3}{2}\right)+f(2)\right)\cdot\tfrac{1}{2} + \tfrac{1}{2}\left(f(2)+f\left(\tfrac{5}{2}\right)\right)\cdot\tfrac{1}{2} + \tfrac{1}{2}\left(f\left(\tfrac{5}{2}\right)+f(3)\right)\cdot\tfrac{1}{2}$$

$$= \tfrac{1}{4}\left(1+\tfrac{3}{4}\right)+\tfrac{1}{4}\left(\tfrac{3}{4}+0\right)+\tfrac{1}{4}\left(0+\left(-\tfrac{5}{4}\right)\right) + \tfrac{1}{4}\left(-\tfrac{5}{4}-3\right)+\tfrac{1}{4}\left(-3-\tfrac{21}{4}\right)+\tfrac{1}{4}\left(-\tfrac{21}{4}-8\right)$$

$$= \tfrac{1}{4}\left(\tfrac{7}{4}\right)+\tfrac{1}{4}\left(\tfrac{3}{4}\right)+\tfrac{1}{4}\left(-\tfrac{5}{4}\right)+\tfrac{1}{4}\left(-\tfrac{17}{4}\right) + \tfrac{1}{4}\left(-\tfrac{33}{4}\right)+\tfrac{1}{4}\left(-\tfrac{53}{4}\right)$$

$$\approx \boxed{-6.125}$$

Note that the actual value of the integral is

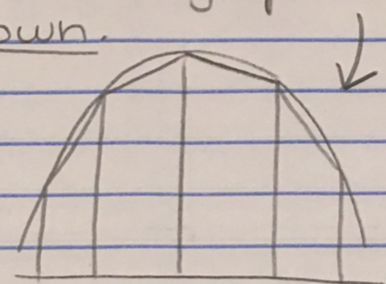
$$\int_0^3 1-x^2\,dx = \left(x-\frac{x^3}{3}\right)\Big|_0^3$$

$$=(3-9)-(0-0)=-6$$

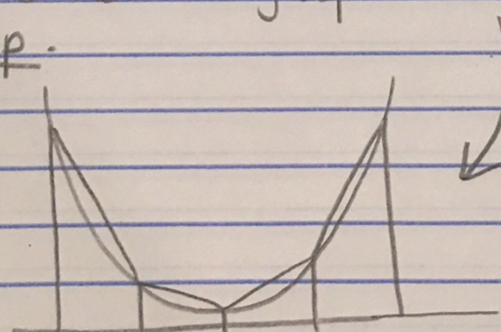
So our trapezoidal rule approximation was actually pretty accurate!

Problem Set 2

- Any of the four approximation methods gives a nonnegative number when f is nonnegative.
- The left endpoint approximation over-approximates the area under the graph of f when f is decreasing.
- The right endpoint approximation over-approximates the area under the graph of f when f is increasing.
- The trapezoidal approximation under-approximates the area under the graph of f when f is concave down.



- The trapezoidal approximation over-approximates the area under the graph of f when f is concave up.



Problem Set 3

4) Every Riemann sum using the minimum value of f in each subinterval gives 0.

Every Riemann sum using the maximum value of f in each subinterval gives 1.

So f is not Riemann integrable!

But as a fun fact - using a more general notion of integration called Lebesgue integration,

$$\int_0^1 f(x)\,dx = 1.$$