Math 1A: Discussion 11/9/18

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November 9, 2018

Problem Set 1

Question 1

Find an antiderivative for the each of the following functions.

- $f(x) = x^2 + 3x + 2$
- $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + e^x$
- $f(x) = \sin(2x) + \cos\left(\frac{x}{2}\right) + \sec^2(x)$

Question 2

Approximate the area under the curve $y = 1 - x^2$ from $x = 0$ to $x = 3$ using 6 subintervals of equal length, by

- Riemann sums, using the left endpoint
- Riemann sums, using the right endpoint
- Riemann sums, using the midpoint
- Trapezoidal rule

Problem Set 2

Question 3

Fill in the blanks below with one of the following words:

- nonnegative, nonzero, nonpositive, increasing, decreasing, concave up, concave down

Here are some facts about approximating areas of functions using Riemann sums.

- Any of the four approximation methods gives a nonnegative number when f is ________________.
- The left endpoint approximation over-approximates the area under the graph of f when f is ________________.
- The right endpoint approximation over-approximates the area under the graph of f when f is ________________.
- The trapezoidal approximation under-approximates the area under the graph of f when f is ________________.
- The trapezoidal approximation over-approximates the area under the graph of f when f is ________________.

Problem Set 3

Question 4 (**)

Consider the function

$$f(x) = 1 \text{ if } x \text{ is irrational}$$

$$f(x) = 0 \text{ if } x \text{ is rational}$$

Recall that a function is Riemann integrable if in the limit as the length of the partitioning subintervals gets smaller and smaller, the maximum value and minimum value Riemann sum approximations approach each other, where for the minimum value approximation, we take the height of the rectangle in each subinterval to be the minimum value of $f(x)$ in that subinterval, and similarly for the maximum value approximation.

Do some Riemann sum approximations using the maximum value and minimum value approximations, to approximate

$$\int_0^1 f(x)dx$$

What do you notice? What can you conclude about the function f?

This function was very important in the development of mathematics, and led mathematicians to extend Riemann integration to a more general notion of integration called Lebesgue integration.