

Discussion 11/28 Solutions

$$1) \int \sin x + \cos x \, dx$$

$$= -\cos x + \sin x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx$$

$$= \arcsin x + C$$

$$\int x + \sqrt{x} + \frac{1}{x} \, dx$$

$$= \frac{1}{2} x^2 + \frac{2}{3} x^{3/2} + \ln|x| + C$$

$$\int 4^x + 5^x + 6^x \, dx$$

$$= \frac{1}{\ln 4} 4^x + \frac{1}{\ln 5} 5^x + \frac{1}{\ln 6} 6^x + C$$

$$2) \int \frac{(\ln x)^5}{x} \, dx = \int u^5 \, du$$

$$u = \ln x$$

$$= \frac{1}{6} u^6 + C$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$= \frac{1}{6} (\ln x)^6 + C$$

$$du = \frac{1}{x} dx$$

$$\text{So } \int_1^e \frac{(\ln x)^5}{x} \, dx = \left(\frac{1}{6} (\ln x)^6 \right) \Big|_1^e$$

$$= \frac{1}{6} (\ln e)^6 - \frac{1}{6} (\ln 1)^6$$

$$= \frac{1}{6} (1)^6 - \frac{1}{6} (0)^6 = \boxed{\frac{1}{6}}$$

$$3) \int \frac{e^x}{\sqrt{1-e^{2x}}} = \int \frac{1}{\sqrt{1-u^2}} du$$

$$u = e^x = \arcsin u + C$$

$$\frac{du}{dx} = e^x = \arcsin(e^x) + C$$

$$du = e^x dx$$

$$4) \int \tan x dx = \int \frac{\sin x}{\cos x} dx$$

$$= \int \frac{1}{u} \cdot (-1) du$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x = \int -\frac{1}{u} du$$

$$du = -\sin x dx = -\ln|u| + C$$

$$-du = \sin x dx = -\ln|\cos x| + C$$

$$5) \int \sin x - \sin x \cos^3 x dx$$

$$= \int \sin x (1 - \cos^3 x) dx$$

$$u = \cos x \quad \frac{du}{dx} = -\sin x \quad du = -\sin x dx$$

$$= \int (1 - u^3) (-1) du$$

$$-du = \sin x dx$$

$$= \int u^3 - 1 du = \frac{u^4}{4} - u + C$$

$$= \frac{\cos^4 x}{4} - \cos x + C$$

$$6) \int \frac{1}{x \ln x \ln(\ln x)} dx = \int \frac{1}{u \ln u} du$$

$$u = \ln x \quad v = \ln u \quad = \int \frac{1}{v} dv$$

$$\frac{du}{dx} = \frac{1}{x} \quad \frac{dv}{du} = \frac{1}{u} \quad = \ln|v| + C$$

$$du = \frac{1}{x} dx \quad dv = \frac{1}{u} du \quad = \ln|\ln u| + C$$

$$7) \int \frac{1}{x^{\frac{1}{2}} + x^{\frac{3}{2}}} dx = \int \frac{1}{\sqrt{x}(1+x)} dx$$

$$= \int \frac{1}{\sqrt{x}(1+(\sqrt{x})^2)} dx$$

$$u = \sqrt{x} \quad = \int \frac{1}{1+u^2} \cdot 2 du$$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad = 2 \arctan u + C$$

$$du = \frac{1}{2\sqrt{x}} dx \quad = 2 \arctan(\sqrt{x}) + C$$

$$\frac{1}{\sqrt{x}} dx = 2 du$$