Math 1B: Discussion 1/31/19

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1 Integration of Rational Functions

Question 1: Warm-up

Find these integrals.

$$\int \frac{1}{6+3x^2}dx$$
$$\int \frac{1}{3-2x}dx$$
$$\int \frac{1}{(3-2x)^2}dx$$

Question 2

Use partial fraction decomposition and/or long division to compute these integrals.

$$\int \frac{1}{x^3-3x^2+2x}dx$$
$$\int \frac{5-x}{(3-x)^2}dx$$
$$\int \frac{4x^2+x+12}{x^3+4x}dx$$
$$\int \frac{1}{1-x^4}dx$$
$$\int \frac{x^4+2}{x^2+2}dx$$

Question 3

Compute

$$\int x^2 \arctan(x)dx$$

(Hint: Use another strategy of integration first.)

Question 4

Compute

$$\int \frac{e^{3x}}{4 - e^{2x}} dx$$

(Hint: Use another strategy of integration first.)

Question 5

Compute

$$\int \frac{\sqrt{x} + 1}{x\sqrt{x} - x + \sqrt{x} - 1} dx$$

(Hint: Let $u = \sqrt{x}$.)

2 Integration of Trigonometric Functions

Reciprocal identities:

$$\csc(x) = \frac{1}{\sin(x)}$$

$$\sec(x) = \frac{1}{\cos(x)}$$

Pythagorean identities:

$$\sin^2(x) + \cos^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) + \csc^2(x)$$

Double angle identities:

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$$

Power reducing identities:

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

Usually, you want to apply these identities to simplify the integrand. Usually the Pythagorean identities (the third, fourth, and fifth identity in the list above) are useful because you can usually do u-substitution after applying them.

Question 6

Calculate

$$\int \cos^2(x)dx$$

two different ways, by (1) integrating by parts, and (2) using the power-reducing rule. Check that you get the same answer both ways. (You may need to use a double angle identity to see that your answers are the same).

Then, calculate

$$\int \cos^4(x)dx$$

(Hint: Write $\cos^4(x) = (\cos^2(x))^2$, and apply the power-reducing formula. After that, use the power-reducing formula again).

Question 7

Calculate

$$\int \sec^4(x)\tan^2(x)dx$$

and

$$\int \sec^3(x)\tan(x)dx$$

(Hint: For one of these, you will do $u = \tan(x)$, and for the other, you will do $u = \sec(x)$. Which one requires which substitution?)

Question 8

Calculate the following integrals involving powers of sine and cosine.

$$\int \sin^5(x)\cos^5(x)dx$$

$$\int \sin^2(x)\cos^2(x)dx$$