Math 1B: Discussion 2/7/19

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Question 1: Approximate Integration

Estimate the integral

$$\int_0^2 (x^2 + x)dx$$

with $n = 4$ using the midpoint rule, left endpoint approximation, right endpoint approximation, trapezoidal rule, and Simpson's rule. Calculate the exact value of the integral. Which approximation works best?

Question 2: Conceptual Reasoning

Fill in the blanks below with one of the following words:

- nonnegative, nonzero, nonpositive, increasing, decreasing, concave up, concave down

Here are some facts about approximating areas of functions using Riemann sums.

- Any of the four approximation methods gives a nonnegative number when f is ________________.
- The left endpoint approximation over-approximates the area under the graph of f when f is ________________.
- The right endpoint approximation over-approximates the area under the graph of f when f is ________________.
- The trapezoidal approximation under-approximates the area under the graph of f when f is ________________.
- The trapezoidal approximation over-approximates the area under the graph of f when f is ________________.

Question 3: Improper Integrals

Evaluate the following improper integrals if they converge, or explain why they diverge.

$$\int_1^2 \frac{1}{\sqrt{x-1}}dx$$

$$\int_1^\infty xe^{-x}dx$$

$$\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} dx$$

$$\int_{-1}^{1} \frac{1}{\sqrt[3]{x}} dx$$

$$\int_{-1}^{1} \frac{1}{x^3} dx$$

$$\int_{0}^{e} (\ln(x))^2 dx$$

$$\int_{0}^{\frac{\pi}{2}} \tan^2(x) dx$$

$$\int_{0}^{\infty} e^{-x} \sin(x) dx$$

(Hint: Squeeze theorem)

Question 4: An Important Example

Let p be a positive number. For what values of p does the improper integral

$$\int_{1}^{\infty} \frac{1}{x^p} dx$$

converge? For what values of p does the improper integral

$$\int_{0}^{1} \frac{1}{x^p} dx$$

converge? (Hint: You can integrate this easily in the case where $p \neq 1$ by the power rule. But something different happens for the case $p = 1$. The integral for $p = 1$ is still easy, just different from the integral for $p \neq 1$. What happens in that case?)

Using your rule above, which of the following integrals converge and which of the following integrals diverge?

$$\int_{1}^{\infty} \frac{1}{\sqrt{x}} dx$$

$$\int_{0}^{1} \frac{1}{x^2} dx$$

$$\int_{0}^{1} \frac{1}{x} dx$$

$$\int_{1}^{\infty} \frac{1}{x} dx$$

$$\int_{1}^{\infty} \frac{1}{x^2\sqrt{x}} dx$$

$$\int_{0}^{1} \frac{1}{\sqrt[4]{x}} dx$$

(Hint: Once you determine the rule from before, you should be able to tell whether these integrals converge or diverge without actually evaluating them.)