Math 1B: Discussion 2/5/19

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1 Integration of Trigonometric Functions

Reciprocal identities:

$$\csc(x) = \frac{1}{\sin(x)}$$
$$\sec(x) = \frac{1}{\cos(x)}$$

Pythagorean identities:

$$\sin^2(x) + \cos^2(x) = 1$$
$$1 + \tan^2(x) = \sec^2(x)$$
$$1 + \cot^2(x) + \csc^2(x)$$

Double angle identities:

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$$

Power reducing identities:

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$
$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

Question 1

Calculate

$$\int \cos^2(x)dx$$

two different ways, by (1) integrating by parts, and (2) using the power-reducing rule. Check that you get the same answer both ways. (You may need to use a double angle identity to see that your answers are the same).

Then, calculate

$$\int \cos^4(x)dx$$

(Hint: Write $\cos^4(x) = (\cos^2(x))^2$, and apply the power-reducing formula. After that, use the power-reducing formula again).

Question 2

Calculate

$$\int \sec^4(x)\tan^2(x)dx$$

$$\int \sec^3(x)\tan(x)dx$$

(Hint: For one of these, you will do $u = \tan(x)$, and for the other, you will do $u = \sec(x)$. Which one requires which substitution?)

Question 3

Calculate the following integrals involving powers of sine and cosine.

$$\int \sin^5(x)\cos^5(x)dx$$

$$\int \sin^2(x)\cos^2(x)dx$$

Question 4 (Tricky but important examples)

Calculate

$$\int \tan(x)dx$$

(Hint: u-substitution.)

$$\int \tan^2(x)dx$$

(Hint: Use a trigonometric identity.)

$$\int \tan^3(x)dx$$

$$\int \tan^4(x)dx$$

(Hint: Use $u = \tan(x)$.)

$$\int \sec(x)dx$$

(Hint: Multiply by $\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}$, and then set $u = \sec(x) + \tan(x)$.)

$$\int \sec^2(x)dx$$

(Hint: Don't overthink it...)

$$\int \sec^3(x)dx$$

(Hint: Integrate by parts with $u = \sec(x)$ and $dv = \sec^2(x)$.)

$$\int \sec^4(x)dx$$

2 Trigonometric Substitution

Question 5

Compute the following integrals.

$$\int \frac{x^2}{(4-x^2)^{3/2}} dx$$

$$\int \frac{e^x}{\sqrt{e^{2x}+2e^x}} dx$$

(Hint: u-substitution, then break the integral into two parts.)

$$\int \frac{2x+1}{(x^2+1)^2} dx$$

(Hint: Break the integral into two parts.)

Question 6

Compute

$$\int \frac{x-2}{x^2+2x+5} dx$$

(Hint: Break the integral up into two parts as $\frac{x-2}{x^2+2x+5} = \frac{x+1}{x^2+2x+5} - \frac{3}{x^2+2x+5}$. Do u-substitution for the first integral. For the second integral, complete the square in the denominator and use trigonometric substitution.)