# Math 1B: Discussion 2/28/19 

Jeffrey Kuan
February 28, 2019

## Question 1: Review of Last Time

Consider

$$
\sum_{n=2}^{\infty} \frac{1}{n^{2}-n}
$$

- Write out a few terms of the series.
- Find the first two partial sums $S_{1}$ and $S_{2}$.
- Do a partial fraction decomposition for

$$
\frac{1}{x^{2}-x}
$$

- Use the partial fraction decomposition to find a formula for $S_{k}$.
- Does the series converge or diverge? This series is called a telescoping series, and you can see why from the previous part.


## Question 2

Do the following series converge or diverge? Use the Integral Test, or the Comparison Test, or both (if possible).

$$
\begin{aligned}
& \sum_{n=1}^{\infty} \frac{1}{4+n^{2}} \\
& \sum_{n=1}^{\infty} \frac{\arctan (n)}{1+n^{2}} \\
& \sum_{n=2}^{\infty} \frac{1}{n^{2}-1} \\
& \sum_{n=0}^{\infty} \frac{n^{3}}{n^{4}-5}
\end{aligned}
$$

$$
\begin{gathered}
\sum_{n=1}^{\infty} \frac{4+\sin \left(n^{2}\right)}{n \sqrt{n}} \\
\sum_{n=1}^{\infty} \frac{1}{n!}
\end{gathered}
$$

(Hint: Can you show that $n!\geq 2^{n-1}$ ?)

## Question 3

Show the following interesting and unexpected result. Prove that

$$
\sum_{n=1}^{\infty} \frac{1}{n}
$$

and

$$
\sum_{n=2}^{\infty} \frac{1}{n \ln (n)}
$$

diverge, yet

$$
\sum_{n=2}^{\infty} \frac{1}{n(\ln (n))^{2}}
$$

converges. (So the extra natural $\log$ in the denominator really makes a difference!)

