Math 1B: Discussion 2/26/19

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February 26, 2019

Question 1

Determine the limits of the following sequences, if they exist.

$$a_n = \frac{1}{n^{1/8}}$$

$$a_n = \frac{(-1)^{n^2}}{\sqrt{n}}$$

$$a_n = (-1)^n 2^n$$

$$a_n = (-1)^n 2^{-n}$$

$$a_n = \cos\left(\frac{1}{n^2}\right)$$

$$a_n = \frac{\sin^3(n^2)}{\sqrt{n}}$$

(Hint: Squeeze Theorem)

Question 2

Use the geometric series formula to find a formula for the partial sums S_k for the following series

$$\sum_{n=1}^{\infty} (-1)^n 3^{-n}$$

Then, calculate

$$\lim_{k\to\infty} S_k$$

to determine whether this series converges or diverges.

Question 3

Do the following series converge or diverge? If the series converges, find its value.

$$\sum_{n=1}^{\infty}(-1)^n$$

$$\sum_{n=1}^{\infty}(-1)^n\left(\frac{2}{3}\right)^n$$

$$\sum_{n=1}^{\infty}(-1)^{n^2}\frac{2n^2}{n^2+4}$$

$$\sum_{n=1}^{\infty}\frac{1}{3^n+2}$$

Question 4

Suppose that you know that $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges. Prove that

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$$

diverges.

Next, suppose you know that $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges. Prove that

$$\sum_{n=1}^{\infty}\frac{1}{n^3}$$

converges. (To do this, use the fact that a sequence that is monotonically increasing and bounded above is convergent).