

Math 1B: Discussion 4/9/19

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Question 1

Solve the following differential equations.

$$\frac{dy}{dx} = \frac{e^x}{\arctan(y) + 2}$$

$$x^2 y' + 4xy' + 5y' = y^2$$

$$e^x \frac{dy}{dx} = y^2 - 4$$

$$\frac{dy}{dx} = ye^{x+y} - e^x \cos(y)$$

Question 2

The following two differential equations are different models for population growth $P(t)$. The first model, given by

$$\frac{dP}{dt} = rP$$

is called **exponential growth**, and the second model, given by

$$\frac{dP}{dt} = rP \left(1 - \frac{P}{K} \right)$$

is called a **logistic model**. In both cases, r is called the **growth factor** and K (which only appears in the logistic model) is called the **limiting capacity**. These are both constants, and $P(t)$ is the function we are solving for.

- What order is each differential equation?
- Use separation of variables to find a general solution to each differential equation.
- For both equations, suppose we know that $r = 2$. For the logistic equation, also suppose that $K = 200$. Given the initial condition, $P(0) = 10$, find a particular solution to each equation. Find $\lim_{t \rightarrow \infty} P(t)$ for both equations. Explain why K is called the **limiting capacity** of the logistic equation.