

Math 1B: Discussion 4/18/19 Solutions

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April 18, 2019

Question 1

$$(2 + 3i) + (1 + 2i) - (-2 - i) = 2 + 3i + 1 + 2i + 2 + i = 5 + 6i$$

$$(1 + i)(2 + i) = 2 + i + 2i - 1 = 1 + 3i$$

$$\frac{1 - i}{2 + 3i} = \frac{1 - i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} = \frac{2 - 3i - 2i - 3}{4 + 9} = -\frac{1}{13} - \frac{5}{13}i$$

Question 2

For $4 - 4i$, we have that $r = 4\sqrt{2}$ and $\theta = \frac{7\pi}{4}$. So we have that

$$4 - 4i = 4\sqrt{2} \left(\cos \left(\frac{7\pi}{4} \right) + i \sin \left(\frac{7\pi}{4} \right) \right)$$

For $-3 - 3\sqrt{3}i$, we have that $r = 6$ and $\theta = \frac{4\pi}{3}$. So we have that

$$-3 - 3\sqrt{3}i = 6 \left(\cos \left(\frac{4\pi}{3} \right) + i \sin \left(\frac{4\pi}{3} \right) \right)$$

For $-\sqrt{3} + i$, we have that $r = 2$ and $\theta = \frac{5\pi}{6}$. So we have that

$$-\sqrt{3} + i = 2 \left(\cos \left(\frac{5\pi}{6} \right) + i \sin \left(\frac{5\pi}{6} \right) \right)$$

For the next complex number, compute that

$$\cos \left(\frac{7\pi}{6} \right) = -\frac{\sqrt{3}}{2}, \quad \sin \left(\frac{7\pi}{6} \right) = -\frac{1}{2}$$

So

$$2 \left(\cos \left(\frac{7\pi}{6} \right) + i \sin \left(\frac{7\pi}{6} \right) \right) = -\sqrt{3} - i$$

For the final complex number, note that

$$\cos \left(\frac{7\pi}{4} \right) = \frac{\sqrt{2}}{2}, \quad \sin \left(\frac{7\pi}{4} \right) = -\frac{\sqrt{2}}{2}$$

So

$$3\sqrt{2} \left(\cos \left(\frac{7\pi}{4} \right) + i \sin \left(\frac{7\pi}{4} \right) \right) = 3 - 3i$$

Question 3

$$\begin{aligned}4 - 4i &= 4\sqrt{2} \left(\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right) \right) = 4e^{\frac{7\pi}{4}i} \\-3 - 3\sqrt{3}i &= 6 \left(\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) \right) = 6e^{\frac{4\pi}{3}i} \\-\sqrt{3} + i &= 2 \left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right) \right) = 2e^{\frac{5\pi}{6}i} \\2 \left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right) \right) &= 2e^{\frac{7\pi}{6}i} \\3\sqrt{2} \left(\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right) \right) &= 3\sqrt{2}e^{\frac{7\pi}{4}i}\end{aligned}$$

Question 4

We have that $1 - \sqrt{3}i = 2e^{\frac{5\pi}{3}i}$. So then

$$\begin{aligned}(1 - \sqrt{3}i)^{40} &= 2^{40}e^{\frac{200\pi}{3}i} = 2^{40}e^{\frac{2\pi}{3}i} = 2^{40} \left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) \right) \\&= 2^{40} \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = -2^{39} + 2^{39}\sqrt{3}i\end{aligned}$$

Next, we have that $-1 - \sqrt{3}i = 2e^{\frac{5\pi}{3}i}$ and $\sqrt{2} + \sqrt{2}i = 2e^{\frac{\pi}{4}i}$. So

$$\frac{-1 - \sqrt{3}i}{\sqrt{2} + \sqrt{2}i} = \frac{2}{2}e^{\frac{5\pi}{3}i - \frac{\pi}{4}i} = e^{\frac{17\pi}{12}i}$$

Then,

$$\left(\frac{-1 - \sqrt{3}i}{\sqrt{2} + \sqrt{2}i} \right)^{100} = e^{100 \cdot \frac{17\pi}{12}i} = e^{\frac{425\pi}{3}i} = e^{\frac{5\pi}{3}i} = \cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

Question 5

We have that

$$\begin{aligned}e^{i\theta} &= \cos(\theta) + i\sin(\theta) \\e^{-i\theta} &= \cos(-\theta) + i\sin(-\theta) = \cos(\theta) - i\sin(\theta)\end{aligned}$$

since $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$ since cosine is even and sine is odd.

Using these two formulas, simple algebra shows that

$$\begin{aligned}\frac{e^{i\theta} - e^{-i\theta}}{2i} &= \frac{1}{2i}(\cos(\theta) + i\sin(\theta) - (\cos(\theta) - i\sin(\theta))) = \sin(\theta) \\\frac{e^{i\theta} + e^{-i\theta}}{2} &= \frac{1}{2}(\cos(\theta) + i\sin(\theta) + \cos(\theta) - i\sin(\theta)) = \cos(\theta)\end{aligned}$$