

# Math 1A: Derive That Derivative!

Jeffrey Kuan

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Team Name: \_\_\_\_\_

Team Member Names: \_\_\_\_\_

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Welcome to America's favorite derivative-based game show, Derive That Derivative! You and your team of four people will have 40 minutes to tackle 20 of the gnarliest derivatives out there. Here are the rules.

- In each question, you will calculate the derivative of the given function, for the number of points specified (4 points, 6 points, 8 points, or 10 points).
- You will be given a score based on the following rubric.
  - Full (100%) credit: Derivative is calculated correctly, or almost completely correctly.
  - Almost complete (75%) credit: Derivative is calculated with a few minor mistakes.
  - Half (50%) credit: Derivative is calculated with several mistakes.
  - Almost no (25%) credit: Derivative is calculated with major mistakes.
  - No (0%) credit: No significant attempt.
- Each question also has a bonus question that is related to the given function.
  - If you answer the bonus question correctly, you get 1.5 times as many points for that question, but ONLY if you got full credit for that derivative.
  - So if Question X is worth 4 points, if you answer Question X and its bonus correctly, you will get a total of 6 points.
  - But if you answer the derivative for Question X incorrectly, even if you get the bonus question for Question X right, you will get no points for the bonus. So it is important to try to do the derivatives correctly, so that you can get the bonus points too.
- The maximum possible score is 180 points.
- Good luck and have fun!

### Question 1 (4 points)

Find the derivative of

$$f(x) = 3x + \frac{2}{\sqrt[3]{x}} + \frac{1}{\sqrt{x}} + e^{-x}$$

**Bonus:** What is  $\lim_{x \rightarrow \infty} f'(x)$ ?

### Question 2 (4 points)

Find the derivative of

$$f(x) = \arctan(x^2 + 1)$$

**Bonus:** What is the domain of  $f(x)$ ?

### Question 3 (4 points)

Find the derivative of

$$f(x) = e^{(e^{-x})} + e^{(-e^x)}$$

**Bonus:** What is  $\lim_{x \rightarrow \infty} f(x)$ ?

### Question 4 (4 points)

Find the derivative of

$$f(x) = 2^{3x} + 3^{2x} + 4^x$$

**Bonus:**  $f'(1)$  can be written as  $\ln(C)$  for some positive integer  $C$ . What is  $C$ ?

### Question 5 (4 points)

Find the derivative of

$$f(x) = \sqrt{e^x + e^{3x} + e^{5x}}$$

**Bonus:** What is the equation of the tangent line to  $f(x)$  at  $x = 0$ ?

### Question 6 (4 points)

Find the derivative of

$$f(x) = \ln(\sqrt{x}) + \ln(x^2) + \ln(x^4) + \ln(x^8)$$

**Bonus:** At what value of  $a$  does the normal line to  $f(x)$  at  $x = a$  have slope  $-1/29$ ?

### Question 7 (4 points)

Find the derivative of

$$f(x) = \arctan\left(\frac{3}{\pi}\arcsin(x)\right)$$

**Bonus:** What is  $f\left(\frac{1}{2}\right)$ ?

### Question 8 (4 points)

Find the derivative of

$$f(x) = \ln(\ln(\ln(x)))$$

**Bonus:** Find a value of  $a$  such that  $f(a) = 4$ .

### Question 9 (6 points)

Find the derivative of

$$f(x) = \frac{\arcsin(e^{-2x})}{x^2 - 1}$$

**Bonus:** What is the domain of  $f(x)$ ?

### Question 10 (6 points)

Find the derivative of

$$f(x) = \frac{\frac{1}{x^2}}{1 + \frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6}}$$

For this question, simplify your answer (no fractions within fractions, simplify the numerator). You can leave the denominator of your derivative as the square of a quantity.

**Bonus:** What is  $\lim_{x \rightarrow \infty} f(x)$ ?

### Question 11 (6 points)

Find the derivative of

$$f(x) = x \sin\left(\frac{1}{x^2}\right) \cos\left(\frac{1}{x^3}\right)$$

**Bonus:** What is  $\lim_{x \rightarrow 0} f(x)$ ?

### Question 12 (6 points)

Find the derivative of

$$f(x) = 3^{\ln(x)} + 4^{\sqrt{x}} + 5^{\sin(x-1)}$$

**Bonus:** What is the slope of the tangent line to  $f$  at  $x = 1$ ?

### Question 13 (6 points)

Find the derivative of

$$f(x) = \sec(\arctan(e^{\tan(x)}))$$

**Bonus:** What is  $f(0)$ ?

### Question 14 (6 points)

Find the derivative of

$$f(x) = \sec(\operatorname{arcsec}(\sin(\arcsin(\tan(\arctan(x^2)))))))$$

**Bonus:** What is  $f'''(10)$ ?



### Question 15 (8 points)

Find the derivative of

$$f(x) = \sqrt{\frac{x^2}{1 + \frac{x}{2x+1}}}$$

**Bonus:** What are the zeros of the function  $f$ ?

### Question 16 (8 points)

Find the derivative of

$$f(x) = \frac{\ln(\ln(x)) + \ln(x)}{e^{\sqrt{x+3}} + \sin(\sqrt{x+3})}$$

**Bonus:** What is  $\lim_{x \rightarrow \infty} f(x)$ ?

### Question 17 (8 points)

Find the derivative of

$$f(x) = \arctan(\arctan(\arctan(x))) + e^{-x}$$

**Bonus:** What is the equation of the tangent line to  $f$  at  $x = 0$ ?

### Question 18 (8 points)

Find the derivative of

$$f(x) = \arctan(\sin(e^x - 1)) + \tan(\ln(\arcsin(x) + 1))$$

**Bonus:** What is  $f(0)$ ?

### Question 19 (10 points)

Find the derivative of

$$f(x) = \sqrt{\arcsin(\ln(x)) + \sqrt{\arccos(\ln(x)) + \sqrt{\arctan(\ln(x))}}}$$

**Bonus:** What is  $f(e)$ ?

### Question 20 (10 points)

Find the derivative of

$$f(x) = e^{\left(\frac{\sin(x)}{x} + \frac{\sin^2(x)}{x^2} + \frac{\sin^3(x)}{x^3}\right)} \cdot \ln\left(1 + \frac{1}{\arctan(e^x)} + \frac{1}{\arctan(e^{2x})}\right)$$

**Bonus:** What is  $\lim_{x \rightarrow 0} f(x)$ , if it exists?