# Midterm 2 Review Session

1) $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)^{\sqrt{x}}$

$$y = \left(\frac{1}{\sqrt{x}}\right)^{\sqrt{x}}$$

$$\ln y = \sqrt{x}\ln\left(\frac{1}{\sqrt{x}}\right)$$

$$\frac{dy}{dx}\frac{1}{y} = \sqrt{x}\,\frac{1}{\frac{1}{\sqrt{x}}}\left(-\frac{1}{2}\frac{1}{x^{3/2}}\right) + \frac{1}{2\sqrt{x}}\ln\left(\frac{1}{\sqrt{x}}\right)$$

$$\frac{dy}{dx}\frac{1}{y} = -\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\ln\left(\frac{1}{\sqrt{x}}\right)$$

$$\frac{dy}{dx} = y\left(-\frac{1}{2\sqrt{x}}\right)\left(1-\ln\left(\frac{1}{\sqrt{x}}\right)\right)$$

$$\boxed{\frac{dy}{dx} = \left(\frac{1}{\sqrt{x}}\right)^{\sqrt{x}}\left(-\frac{1}{2\sqrt{x}}\right)\left(1-\ln\left(\frac{1}{\sqrt{x}}\right)\right)}$$

$\frac{d}{dx}(2+\cos x)^{\sin x}$

$$y = (2+\cos x)^{\sin x}$$

$$\ln y = \sin x \ln(2+\cos x)$$

$$\frac{dy}{dx}\frac{1}{y} = \cos x \ln(2+\cos x) + \frac{\sin x}{2+\cos x}(-\sin x)$$

$$\frac{dy}{dx} = y\left(\cos x\ln(2+\cos x) - \frac{\sin^2 x}{2+\cos x}\right)$$

$$\boxed{\frac{dy}{dx} = (2+\cos x)^{\sin x}\left(\cos x\ln(2+\cos x) - \frac{\sin^2 x}{2+\cos x}\right)}$$

2) $\arctan(\sqrt{xy}) - \frac{\pi}{4}x^2y = 0$

Implicit differentiation

$$\frac{1}{1+(\sqrt{xy})^2}\left(\frac{1}{2\sqrt{xy}}\right)\left(x\frac{dy}{dx}+y\right) - \frac{\pi}{2}xy - \frac{\pi}{4}x^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(\frac{x}{2\sqrt{xy}(1+xy)} - \frac{\pi}{4}x^2\right)$$

$$= -\frac{y}{2\sqrt{xy}(1+xy)} + \frac{\pi}{2}xy$$

Plug in (1, 1)

$$\frac{dy}{dx}\left(\frac{1}{4} - \frac{\pi}{4}\right) = -\frac{1}{4} + \frac{\pi}{2}$$

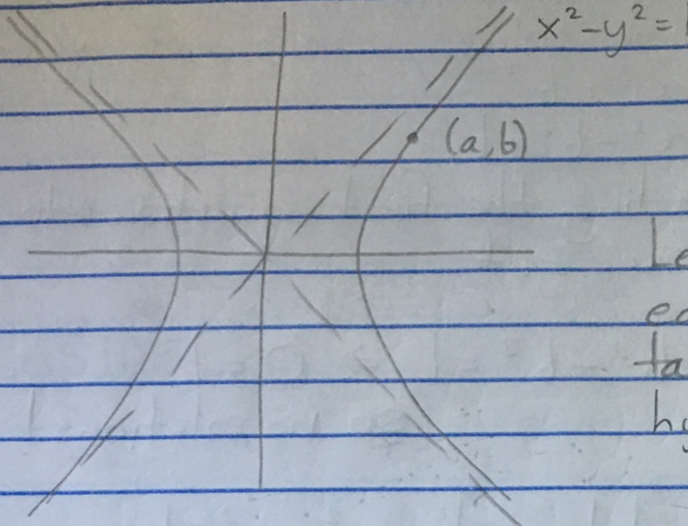
$$\frac{dy}{dx} = \frac{-\frac{1}{4}+\frac{\pi}{2}}{\frac{1}{4}-\frac{\pi}{4}} = \frac{2\pi - 1}{1-\pi}$$

Tangent line: $y - 1 = \frac{2\pi-1}{1-\pi}(x-1)$

Normal line: $y - 1 = \frac{\pi - 1}{2\pi - 1}(x-1)$

(Slope of normal line is the negative reciprocal of the slope of the tangent line)

3)




Let us write the equation of a general tangent line to the hyperbola at $(a, b)$.

Find the slope:

$$x^2-y^2=1$$
$$2x-2y\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=\frac{x}{y}, \quad \text{so } \left.\frac{dy}{dx}\right|_{(a,b)}=\frac{a}{b}.$$

So the tangent line to the hyperbola at $(a,b)$ is

$$y-b=\frac{a}{b}(x-a).$$

So for the tangent line to pass through the origin, we must have

$$0-b=\frac{a}{b}(0-a)$$
$$\Rightarrow a^2=b^2$$

Since $(a,b)$ is on $x^2-y^2=1$, we must also have $a^2-b^2=1$.

$\Rightarrow$

So we want to solve

$$\begin{cases} a^2 = b^2 \\ a^2 - b^2 = 1. \end{cases}$$

But substituting the first equation into the second, we get

$$b^2 - b^2 = 1 \Rightarrow 0 = 1$$

so no solutions!

* So no tangent lines to $x^2 - y^2 = 1$ pass through the origin.

4) $\lim_{x\to\infty} (e^x - x^2 - \ln x)$ $\quad \infty - \infty - \infty$ indeterminate difference

Factor out the fastest growing term, which is $e^x$.

$$= \lim_{x\to\infty} e^x \Big(1 - \underbrace{\frac{x^2}{e^x}}_{①} - \underbrace{\frac{\ln x}{e^x}}_{②}\Big) = \boxed{\infty}$$

$(\infty \cdot (1 - 0 - 0))$

see below

ASIDE:

① $\lim_{x\to\infty} \frac{x^2}{e^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0.$

$\frac{\infty}{\infty} \qquad \frac{\infty}{\infty} \qquad \frac{2}{\infty} = 0$

② $\lim_{x\to\infty} \frac{\ln x}{e^x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{e^x} = 0.$

$\frac{0}{\infty} = 0$

5) $\lim\limits_{x \to 1} \dfrac{e^x - e}{x^2 - 1} = \lim\limits_{x \to 1} \dfrac{e^x}{2x} = \boxed{\dfrac{e}{2}}$

$\frac{0}{0}$

$\infty^0$

$\lim\limits_{x \to \infty} (e^{2x} + 1)^{\frac{1}{x}} = \boxed{e^2}$

$\Downarrow$

ASIDE:

$$\lim_{x \to \infty} \ln (e^{2x} + 1)^{\frac{1}{x}}$$

$$= \lim_{x \to \infty} \frac{1}{x} \ln (e^{2x} + 1)$$

$0 \cdot \infty$

L'H

$$= \lim_{x \to \infty} \frac{\ln (e^{2x} + 1)}{x} \stackrel{\text{L'H}}{=} \lim_{x \to \infty} \frac{\frac{2e^{2x}}{e^{2x} + 1}}{1}$$

$\frac{\infty}{\infty}$

$$= \lim_{x \to \infty} \frac{2e^{2x}}{e^{2x} + 1}$$

$\frac{\infty}{\infty}$

L'H

$$= \lim_{x \to \infty} \frac{4e^{2x}}{2e^{2x}} = \lim_{x \to \infty} 2 = \underline{2}$$

6) $\dfrac{d}{dx} \left(3 \sec^2 (\arcsin(x^2))\right)$

$$= 3 \frac{d}{dx} \left( \sec^2 (\arcsin(x^2)) \right)$$

same as

$$= 3 \frac{d}{dx} \left[ \left( \sec (\arcsin (x^2)) \right)^2 \right]$$

$$= 3 \cdot 2 \sec(\arcsin(x^2)) \cdot \sec(\arcsin(x^2)) \cdot \tan(\arcsin(x^2)$$

$$\cdot \frac{1}{\sqrt{1 - (x^2)^2}} \cdot 2x$$

$$\frac{d}{dx}\left(\log_2\left(\log_3\left(\csc(x)\right)\right)\right)$$

$$= \frac{1}{(\ln 2)(\log_3(\csc x))} \frac{1}{(\ln 3)\csc x} (-\csc x \cot x)$$

<u>Remember:</u> $\frac{d}{dx}(\log_a x) = \frac{1}{(\ln a)\cdot x}$

and $\frac{d}{dx}(a^x) = (\ln a)\cdot a^x$.

$$\frac{d}{dx}\left(\operatorname{arccot}\left(\sqrt{1-\sqrt[3]{x}}\right)\right)$$

$$= -\frac{1}{1+\left(\sqrt{1-\sqrt[3]{x}}\right)^2} \frac{1}{2\sqrt{1-\sqrt[3]{x}}} \left(-\frac{1}{3}x^{-2/3}\right)$$

$$= \frac{1}{6} \frac{1}{2-\sqrt[3]{x}} \frac{1}{x^{2/3}\left(\sqrt{1-\sqrt[3]{x}}\right)}$$

7) $f(x)=\frac{1}{7}x^{\frac{7}{3}}-x^{\frac{1}{3}}$

$=\frac{1}{7}x^{\frac{1}{3}}(x^{2}-7)$ zeros: $0, \pm\sqrt{7}$

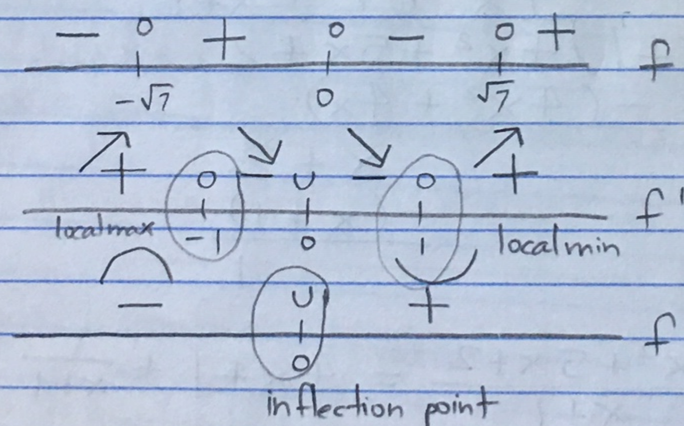
$f'(x)=\frac{1}{3}x^{4/3}-\frac{1}{3}x^{-2/3}$

$=\frac{1}{3}x^{-2/3}(x^{2}-1)$

$=\frac{1}{3}x^{-2/3}(x-1)(x+1)$ zeros: $-1, 1$
UND at 0

$f''(x)=\frac{4}{9}x^{1/3}+\frac{2}{9}x^{-5/3}$

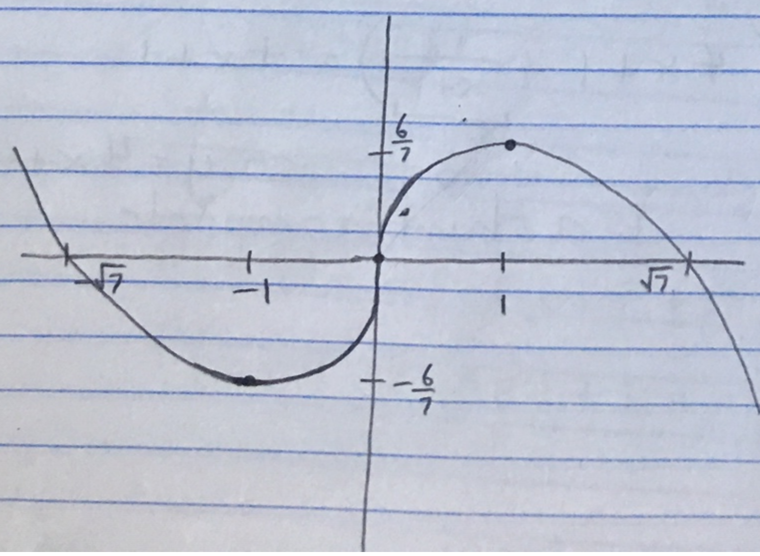
$=\frac{2}{9}x^{-5/3}(2x^{2}+1)$ UND at $x=0$.




$f(1)=-\frac{6}{7}$ (local min)

$f(-1)=\frac{6}{7}$ (local max)

$f(0)=0$ (inf point)



$$f(x) = \frac{4x^3+5x^2+2x}{x^2+x}$$

$$= \frac{4x^2+5x+2}{x+1} \quad \text{when } x \neq 0$$

UND at $x=0$

hole at $x=0$

VA: $x=-1$

HA: $\lim\limits_{x\to\infty} \frac{4x^2+5x+2}{x+1} \left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)$

$$= \lim_{x\to\infty} \frac{4+\frac{5}{x}+\frac{2}{x^2}}{\frac{1}{x}+\frac{1}{x^2}} = \text{infinite} \Rightarrow \text{No HA}$$

$$\frac{4}{0}$$

Slant:

$$\begin{array}{r} 4x+1+\frac{1}{x+1} \\ x+1 \overline{)\, 4x^2+5x+2} \\ -(4x^2+4x) \\ \hline x+2 \\ -(x+1) \\ \hline 1 \end{array}$$

So $\frac{4x^2+5x+2}{x+1} = 4x+1+\frac{1}{x+1} = f$

$$\lim_{x\to\infty}\left(4x+1+\underbrace{\frac{1}{x+1}}_{\downarrow\, 0}\right) = 4x+1$$

so $y=4x+1$ is a slant asymptote

We can use the Quotient Rule to calculate $f'$, $f''$, or note that from long division, we have that

$$f = 4x + 1 + \frac{1}{x+1} = \frac{4x^2+5x+2}{x+1}$$

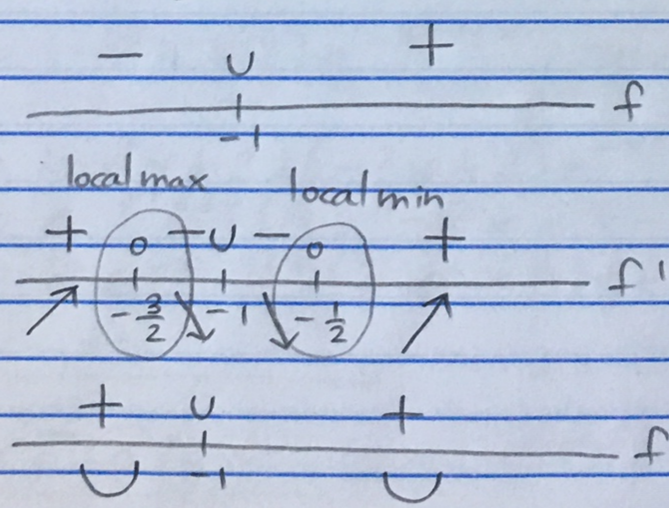
So $$f' = 4 - \frac{1}{(x+1)^2} = \frac{4x^2+8x+4-1}{(x+1)^2}$$

$$= \frac{4x^2+8x+3}{(x+1)^2} = \frac{(2x+3)(2x+1)}{(x+1)^2}$$

$$f'' = \frac{2}{(x+1)^3}$$

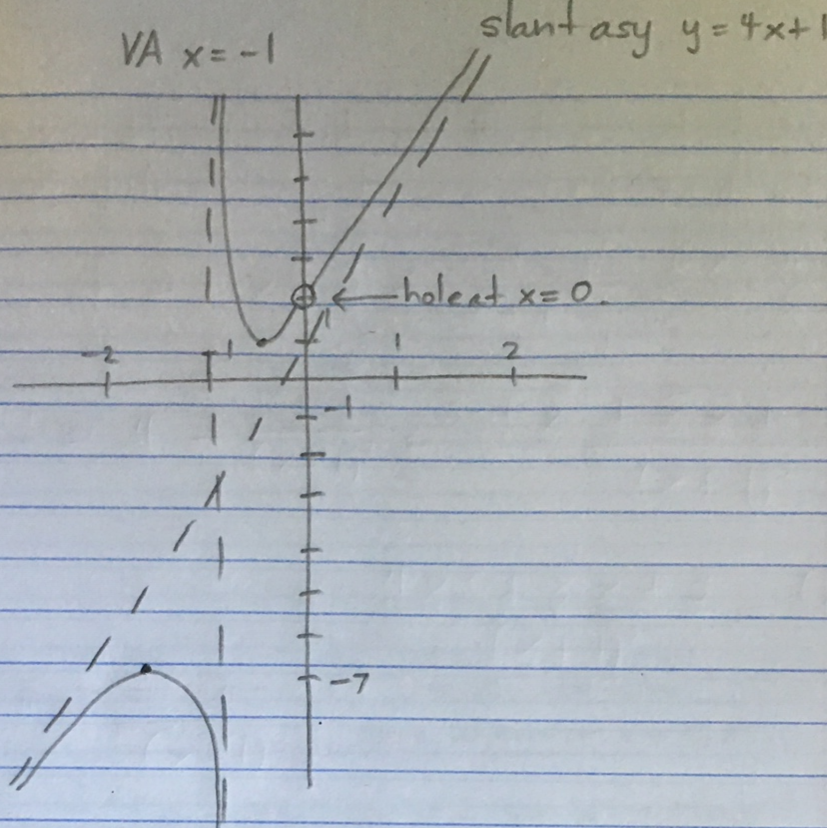
zeros of $f$: $4x^2+5x+2=0$

$$x = \frac{-5 \pm \sqrt{25-32}}{8} \leftarrow \text{no real zeros!}$$




$$f\left(-\tfrac{3}{2}\right) = 4\left(-\tfrac{3}{2}\right) + 1 + \frac{1}{\left(-\frac{3}{2}\right)+1} = -7$$

$$f\left(-\tfrac{1}{2}\right) = 4\left(-\tfrac{1}{2}\right) + 1 + \frac{1}{\left(-\frac{1}{2}\right)+1} = 1$$




$\lim_{x\to -\infty} f(x) = 0$

$f(x) = e^{-x^2+2x}$ no zeros $* \lim_{x\to\infty} f(x) = 0$ so

HA at $y=0$

$f'(x) = (-2x+2)e^{-x^2+2x}$ zero at $x=1$

$$f''(x) = (-2x+2)(-2x+2)e^{-x^2+2x} - 2e^{-x^2+2x}$$
$$= e^{-x^2+2x}(4x^2-8x+4-2)$$
$$= 2e^{-x^2+2x}(2x^2-4x+1)$$

$+$ f

$+ \nearrow$ 0 $\searrow -$ f'

1 local max

$+$ 0 $-$ 0 $+$ f''

$1-\frac{\sqrt{2}}{2}$ $1+\frac{\sqrt{2}}{2}$

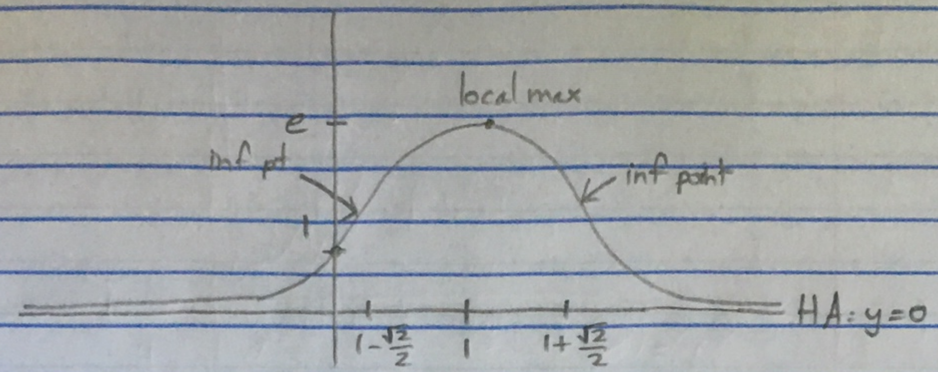
inf. pts

$$x = \frac{4 \pm \sqrt{16-8}}{2\cdot 2}$$
$$= 1 \pm \frac{\sqrt{8}}{4} = 1 \pm \frac{\sqrt{2}}{2}$$

$f(1) = e^1 = e$

local max

$f(0) = 1$




$\frac{0}{0}$ at $x = \pi, 3\pi, \ldots, -\pi, -3\pi, \ldots$

$$f(x) = \frac{\sin x}{1+\cos x}$$

$\lim_{x\to\pi} \frac{\sin x}{1+\cos x} = \lim_{x\to\pi} \frac{\cos x}{-\sin x}$ = infinite So VA at $x = \pi + 2\pi k$

zero whenever $x = 0, 2\pi, \ldots, -2\pi, \ldots$

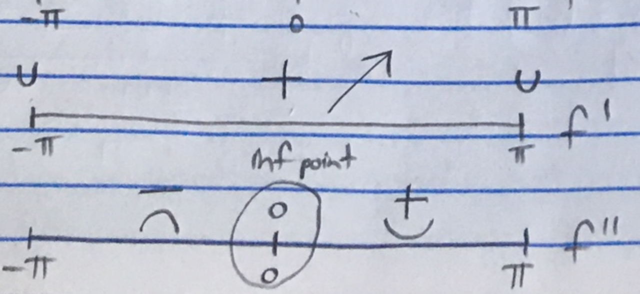
$$f'(x) = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} \qquad \sin^2 x + \cos^2 x = 1$$

$$= \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

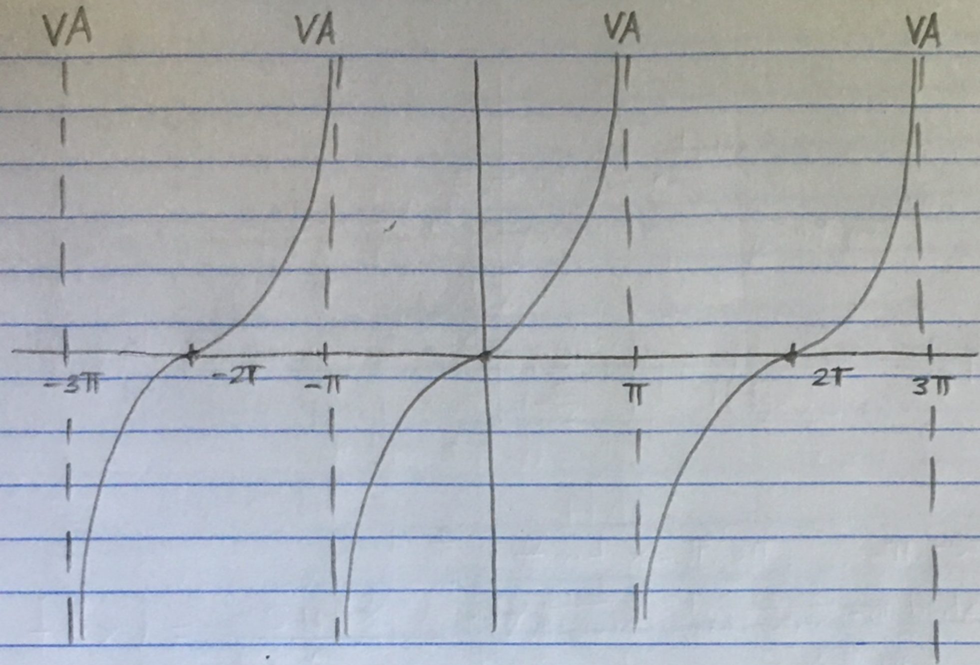
$$f''(x) = -\frac{1}{(1+\cos x)^2}(-\sin x) = \frac{\sin x}{(1+\cos x)^2}$$

This has period $2\pi$ so we just need to understand what happens on $[-\pi, \pi]$.




$f(0) = 0$

inf. point




$f(x) = x^{\frac{1}{x}}$ for $x>0$

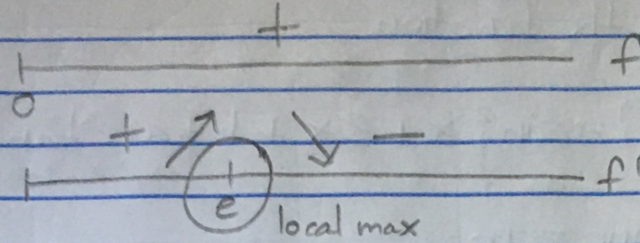
$$y = x^{\frac{1}{x}}$$
$$\ln y = \frac{1}{x}\ln x$$
$$\frac{1}{y}y' = \frac{1}{x^2} + \left(-\frac{1}{x^2}\ln x\right) = \frac{1}{x^2}(1-\ln x)$$
$$y' = \frac{x^{\frac{1}{x}}}{x^2}(1-\ln x) = x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right)$$
$$\left(x^{\frac{1}{x}}\right)' = x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right)$$

so $$y'' = x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right)^2 + x^{\frac{1}{x}}\left(\frac{x^2\left(-\frac{1}{x}\right)-(1-\ln x)(2x)}{x^4}\right)$$
$$= x^{\frac{1}{x}}\left(\frac{1-2\ln x+(\ln x)^2 - x - 2x + 2x\ln x}{x^4}\right)$$
$$= x^{\frac{1}{x}}\left(\frac{1-2\ln x+2x\ln x+(\ln x)^2-3x}{x^4}\right)$$

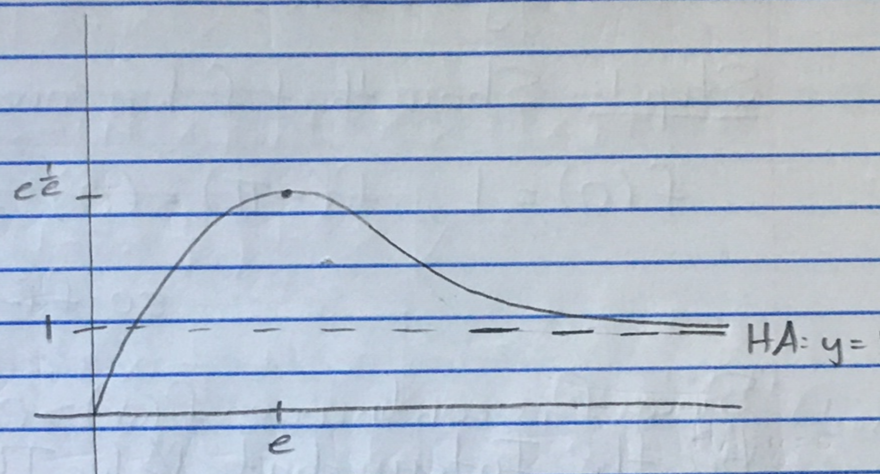
Too hard to work with $y''$, so just use $y$ and $y'$ to curve sketch.

f no zeros for $x > 0$

f' is zero at $1 - \ln x = 0 \Rightarrow x = e$




$f(e) = e^{\frac{1}{e}}$




$\infty^0$

Note: $\lim_{x \to \infty} x^{\frac{1}{x}} = e^0 = 1$

ASIDE $\lim_{x \to \infty} \ln\left(x^{\frac{1}{x}}\right)$

$= \lim_{x \to \infty} \frac{1}{x} \ln(x) = \lim_{x \to \infty} \frac{\ln x}{x}$

$\frac{\infty}{\infty}$

$= \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = 0$

$e^{-\infty} = 0$

$\lim_{x \to 0^+} x^{\frac{1}{x}} = 0$

ASIDE $\lim_{x \to 0^+} \ln\left(x^{\frac{1}{x}}\right) = \lim_{x \to 0^+} \frac{\ln x}{x} = -\infty$

$\frac{-\infty}{0}$

8) Showing that

$$\sin^2 x + 3x + 1 = -\arctan x$$

has exactly one solution is the same as showing that the function

$$f(x) = \sin^2 x + 3x + 1 + \arctan x$$

has exactly one zero.

Step 1: Show that f has one zero.

$$f(0) = 1, \quad f(-\pi) = (0)^2 + \underbrace{(-3\pi) + 1}_{<0} + \underbrace{\arctan(-\pi)}_{<0}$$

so $f(-\pi) < 0$.

Since f is continuous, $f(0) > 0$, and $f(-\pi) < 0$, by the Intermediate Value Theorem, f has a zero between $x = -\pi$ and $x = 0$.

Step 2: Show that f does not have two zeros.

Use proof by contradiction. Suppose f two zeros, call them a and b. Then

$$f(a) = f(b) = 0.$$

Since f is continuous and differentiable, there is a c between a and b such that $f'(c) = 0$ by Rolle's Theorem.

But $f'(x) = 2\sin x\cos x + 3 + \frac{1}{1+x^2}$.

$$\left.\begin{array}{l} -1 \le \sin x \le 1 \\ -1 \le \cos x \le 1 \end{array}\right] \rightarrow -1 \le \sin x\cos x \le 1$$

$$\rightarrow -2 \le 2\sin x\cos x \le 2$$

$\frac{1}{1+x^2} > 0$. So

$$f'(x) = 2\sin x\cos x + 3 + \frac{1}{1+x^2}$$

$$\geq -2 + 3 + 0 = 1 > 0.$$

So $f'(x) > 0$, which contradicts that $f'(c) = 0$. So by contradiction, $f$ does not have two zeros.

Since $f$ has one zero, but does not have two zeros, $f$ has exactly one zero.

9) Showing that

$$\cos x - 2x^2 = c$$

has at most two real solutions is equivalent to showing that

$$f(x) = \cos x - 2x^2 - c$$

has at most two zeros. So we must show that $f$ does not have three zeros.

Use proof by contradiction. Suppose that $f$ has three zeros. Call them $p, q, r$ with $p < q < r$.

Then, $f(p) = f(q) = 0$, $f(q) = f(r) = 0$. By Rolle's theorem (since $f$ is continuous and differentiable), there exist $a$ and $b$ with $p < a < q$ and $q < b < r$ such that $f'(a) = 0$ and $f'(b) = 0$.

→

Then, $f' = -\sin x - 4x$ is also continuous and differentiable, so since $f'(a) = f'(b) = 0$, by using Rolle's theorem for $f'$, we have that there exists $d$ with $a < d < b$ such that

$$f''(d) = 0.$$

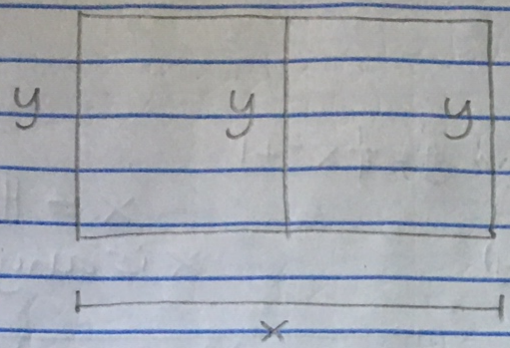
But $$f''(x) = -\underbrace{\cos x}_{-1 \le \ \le 1} - 4 \le -3 < 0.$$

So $f''$ is never zero, which contradicts that $f''(d) = 0$.

So by contradiction, $f$ cannot have three zeros. So $f$ has at most two zeros.

(Note: We did not need to use IVT first since we only need to show that $f$ has <u>at most</u> two zeros, not <u>exactly</u> two zeros.)

10)



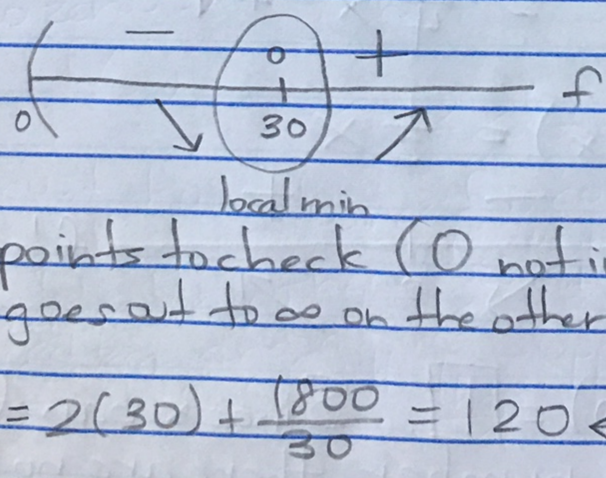

$f(x) = 2x + 3y$ (amount of fencing needed)

$xy = 600 \Rightarrow y = \frac{600}{x}$

$f(x) = 2x + \frac{1800}{x}$   $x > 0$   ($x \neq 0$ since $xy = 600$)

$f'(x) = 2 - \frac{1800}{x^2} = \frac{2x^2 - 1800}{x^2}$

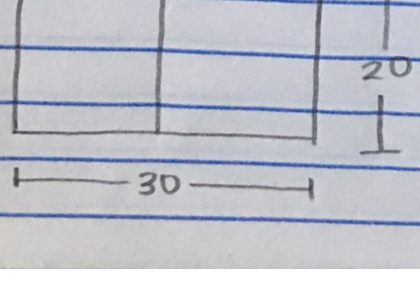
$x = \pm 30$ zeros




no endpoints to check (0 not in domain and domain goes out to ∞ on the other end)

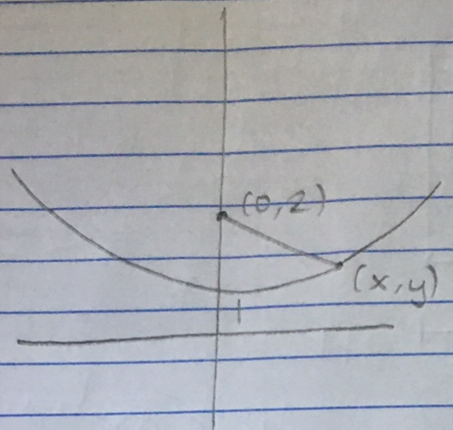
$f(30) = 2(30) + \frac{1800}{30} = 120 \leftarrow$ absolute min

ANS:




is the playground that requires the least fencing.

11)




$y = \frac{1}{4}x^2 + 1$

$x \in \mathbb{R}$

x is any real number

$\sqrt{(x-0)^2 + (y-2)^2} = \text{dist}$

$$f(x) = \sqrt{x^2 + \left(\left(\tfrac{1}{4}x^2 + 1\right) - 2\right)^2}$$

$$= \sqrt{x^2 + \left(\tfrac{1}{4}x^2 - 1\right)^2}$$

OR can just find f' here,

$$f' = \frac{2x + 2\left(\frac{1}{4}x^2 - 1\right)\left(\frac{1}{2}x\right)}{2\sqrt{x^2 + \left(\frac{1}{4}x^2 - 1\right)^2}}$$

$$= \frac{x\left(2 + \frac{1}{4}x^2 - 1\right)}{2\sqrt{x^2 + \left(\frac{1}{4}x^2 - 1\right)^2}}$$

$$= \frac{x(x^2+4)}{8\sqrt{x^2 + \left(\frac{1}{4}x^2 - 1\right)^2}}$$

$$= \sqrt{x^2 + \frac{1}{16}x^4 - \frac{1}{2}x^2 + 1}$$

$$= \sqrt{\frac{1}{16}x^4 + \frac{1}{2}x^2 + 1}$$

OR can simplify

$$= \frac{1}{4}x^2 + 1$$

$$f'(x) = \frac{1}{2}x$$

$-$     $+$

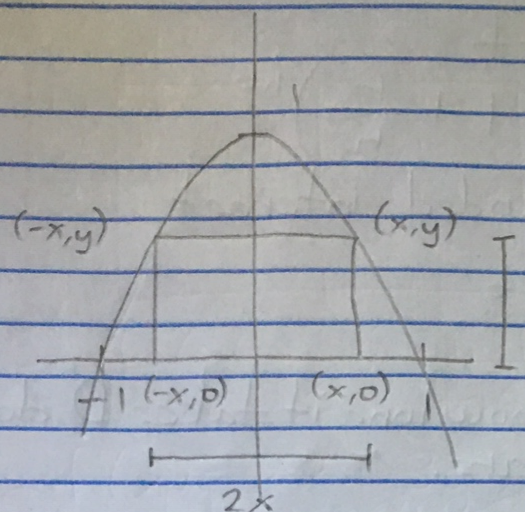
—— 0 —— f'

↘    ↗

$f(0) = \boxed{1}$ Since x can be any real number, there are no endpoints to check.

ABS MIN

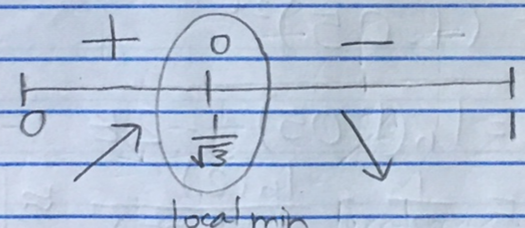
12)




$0 \le x \le 1$

$f(x) = 2x(1-x^2)$
$= 2x - 2x^3$

$f'(x) = 2 - 6x^2$
$= 2(1-3x^2)$




$f(1) = 0$
$f(0) = 0$

$f\left(\frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}\left(1 - \frac{1}{3}\right) = \underbrace{\frac{4}{3\sqrt{3}}}_{\text{abs max}}$

$\boxed{\begin{array}{l} \text{length is } 2\left(\frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \\ \text{width is } 1 - \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{2}{3} \end{array}}$

$(2x)$

$(y = 1 - x^2)$

13) compounded interest

$$P = P_0\left(1+\frac{r}{n}\right)^{nt}$$

continuously compounded interest

$$P = P_0 e^{rt}$$

We want to find how long it takes D dollars to become 2D dollars.

① 10% semiannually

$$2D = D\left(1+\frac{0.10}{2}\right)^{2t}$$

$$2 = (1.05)^{2t}$$

$$\ln 2 = \ln(1.05) \cdot 2t$$

$$t = \frac{1}{2} \cdot \frac{\ln(2)}{\ln(1.05)} \approx \boxed{7.10 \text{ years}}$$

② 10% continuously

$$2D = De^{0.10t}$$

$$2 = e^{0.10t}$$

$$\ln 2 = 0.10t$$

$$t = 10\ln 2 \approx \boxed{6.93 \text{ years}}$$

14) Apply Newton's Law of Cooling to the bread and butter individually.

$$T(t) = T_A + (T_0 - T_A)e^{-kt}$$

Let $T_{bread}(t)$ and $T_{butter}(t)$ be the temperature of the bread and butter at time $t$.

$$T_{bread}(t) = 25 + (125 - 25)e^{-kt}$$
$$= 25 + 100e^{-kt}$$
$$T_{bread}\left(\tfrac{1}{2}\right) = 84.$$

So $84 = 25 + 100e^{-k \cdot \frac{1}{2}}$

$$\frac{59}{100} = e^{-\frac{k}{2}}$$

$$\ln\left(\frac{59}{100}\right) = -\frac{k}{2} \qquad k = -2\ln\left(\frac{59}{100}\right)$$
$$= 2\ln\left(\frac{100}{59}\right)$$

Thus, $T_{bread}(t) = 25 + 100e^{-2\ln\left(\frac{100}{59}\right)t}$

$$T_{butter}(t) = 25 + (5 - 25)e^{-kt}$$
$$= 25 - 20e^{-kt}$$

$T_{butter}\left(\tfrac{1}{2}\right) = 20$, so $20 = 25 - 20e^{-k \cdot \frac{1}{2}}$

$$\Rightarrow \frac{1}{4} = e^{-\frac{k}{2}}$$
$$-\frac{k}{2} = \ln\left(\frac{1}{4}\right)$$
$$k = -2\ln\left(\frac{1}{4}\right) = 2\ln 4.$$

So $T_{butter}(t) = 25 - 20e^{-2(\ln 4) \cdot t}$.

Thus, $T(t) = T_{bread}(t) - T_{butter}(t)$

$$= \left(25 + 100e^{-2\ln\left(\frac{100}{59}\right)t}\right) - \left(25 - 20e^{-2(\ln 4) \cdot t}\right)$$
$$= 100e^{-2\ln\left(\frac{100}{59}\right)t} + 20e^{-2\ln(4)t}.$$