Question 2:

Evaluate the following expression. Answer in the simplest form. (2 points each)

(2.1) $\log_5(25\sqrt{5})$

$= \log_5(25 \cdot \sqrt{5}) = \log_5(25) + \log_5(\sqrt{5})$

$= 2 + \frac{1}{2}$

$= 5/2$

(2.2) $\log \frac{1}{10000}$

$\log_{10}\left(\frac{1}{10{,}000}\right) = \log_{10}(10^{-4}) = -4$

(2.3) $81^{\frac{3}{4}}$

$81 = 3^4 \qquad (81^{3/4}) = (3^4)^{3/4} = 3^3 = 27$

(2.4) $\log_8 2^{6.3}$

$\log_8 2^{6.3} = \dfrac{\log_2 2^{6.3}}{\log_2 8} = \dfrac{6.3}{3} = 2.1$

(2.5) $\left(\frac{3}{2}\right)^{-3}$

$= \dfrac{3^{-3}}{2^{-3}} = \dfrac{1/27}{1/8} = 8/27$

Question 4:

Let x, y, v, w be real numbers such that

$$\log_4 x = 1.35, \qquad \log_4 y = -0.2,$$
$$\ln v = 2.8, \qquad \ln w = 0.4.$$

Evaluate the following (2 points each).

(4.1) $\log_4\left(\frac{xy}{4}\right)$

$$\log_4 x + \log_4 y - \log_4 4 = 1.35 - 0.2 - 1$$
$$= 1.15 - 1 = .15$$

(4.2) $\log_4(x^{100})$

$$= 100 \cdot \log_4(x) = 100 \cdot 1.35 = 135$$

(4.3) $\log_w v$

$$= \frac{\ln v}{\ln w} = \frac{2.8}{.4} = 7$$

(4.4) $\log_{16} y$

$$= \frac{\log_4 y}{\log_4 16} = \frac{-.2}{2} = -.1$$

(4.5) $\frac{v}{w^2}$

$$= e^{\ln(v/w^2)} = e^{\ln v - 2\ln w} = e^{2.8 - .8}$$
$$= e^2 .$$

Question 5:
Let f be the function defined by

$$f(x) = 3 \cdot e^{5x}.$$

(5.1) (5 points) Find a formula for f^{-1}.

$$y = 3 \cdot e^{5x}$$

$$\frac{y}{3} = e^{5x}$$

$$\ln\left(\frac{y}{3}\right) = 5x$$

$$x = \frac{1}{5} \ln\left(\frac{y}{3}\right)$$

(5.2) (5 points) Let g be the function defined by

$$g(x) = \ln f(x).$$

The graph of g is a line. Find the slope and the y-intercept of the graph of g.

$$\ln\left(3 \cdot e^{5x}\right)$$

$$= \ln 3 + \ln e^{5x}$$

$$= \ln 3 + 5x$$

Slope $= 5$ $\quad$ y-int $= \ln 3$.

Question 6:

A colony of bacteria is growing exponentially, doubling in size every 100 minutes. At time $t = 0$, the number of bacteria is 300.

(6.1) (5 points) Find the function f such that $f(t)$ is equal to the number of bacteria at time t, where the unit of t is minutes.

$$A(t) = A(0) \cdot 2^{t/d}$$

doubling time

$$= 300 \cdot 2^{t/100}$$

(6.2) (5 points) For this problem, use the approximations $\log 2 \approx 0.3$ and $\log 3 \approx 0.5$. Approximately how many minutes will it take for the number of bacteria to be 1800?

$$1800 = 300 \cdot 2^{t/100}$$

$$6 = 2^{t/100}$$

$$\log_{10}(6) = \frac{t}{100} \log_{10}(2)$$

$$\frac{\log_{10}(6)}{\log_{10}(2)} = \frac{t}{100}$$

$$t = 100 \cdot \frac{\log_{10}(6)}{\log_{10}(2)}$$

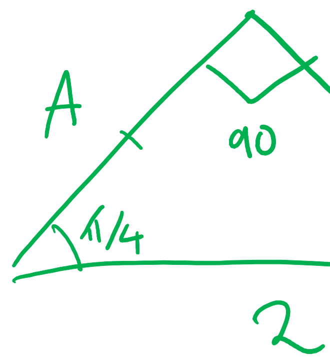
$$= 100 \cdot \frac{\log_{10}(3 \cdot 2)}{\log_{10}(2)}$$

$$= 100 \cdot \frac{\log_{10}(3) + \log_{10}(2)}{\log_{10}(2)}$$

$$= 100 \cdot \frac{.3 + .5}{.3} = 800/3$$

deceptive: this triangle is isoceles.

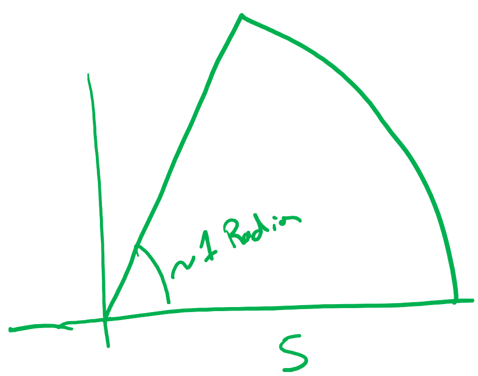
(5) (a) Find the length of a side of a right triangle that has a hypotenuse of length 2 and an angle of $\frac{\pi}{4}$ radians.



$$\frac{A}{2} = \cos \pi/4 = \sqrt{2}/2$$

$$A = \sqrt{2}.$$

(b) Find the area of a 1 radian slice in a circle with radius 5.



$$\text{Area} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \cdot (5)^2 \cdot 1$$

$$= 25/2.$$

(c) Calculate $\sin \frac{\pi}{12}$ given that $\cos \frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$. Please simplify your answer.
hint: $8 - 4\sqrt{3} = \left(\sqrt{6} - \sqrt{2}\right)^2$

$$\left(\cos \frac{\pi}{12}\right)^2 + \left(\sin \frac{\pi}{12}\right)^2 = 1$$

$$\frac{\sqrt{6}+\sqrt{2}}{4} + \left(\sin \frac{\pi}{12}\right)^2 = 1$$

$$\left(\sin \frac{\pi}{12}\right)^2 = 1 - \frac{(\sqrt{6}+\sqrt{2})^2}{4}$$

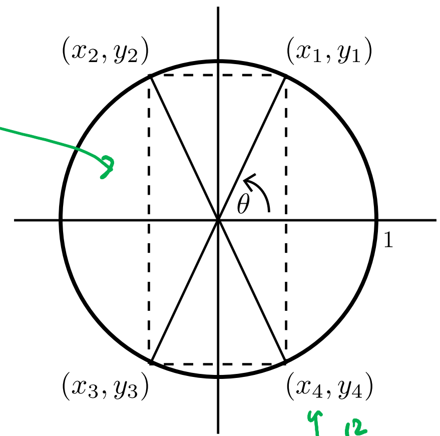
$$\boxed{\left(\sin \frac{\pi}{12}\right) = +\sqrt{1 - \frac{(\sqrt{6}+\sqrt{2})^2}{4}}}$$

positive because $\pi/12$ in quadrant I.

Question 5:

The following figure is the unit circle centered at the origin. The angle between the positive x-axis and the line passing through the origin and the point (x_1, y_1) is equal to θ. The line passing through (x_1, y_1) and (x_2, y_2) and the line passing through (x_3, y_3) and (x_4, y_4) are parallel to the x-axis. The line passing through (x_2, y_2) and (x_3, y_3) and the line passing through (x_4, y_4) and (x_1, y_1) are parallel to the y-axis.

Really not drawn to scale! Think that $\frac{12}{13} \sim 1$.



$\frac{12}{13}$

$$y = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{25/169} = 5/13.$$

Given that $x_4 = \frac{12}{13}$. Evaluate the following. You do not need to show work. (1 point each)

(5.1) x_1 $= x_4 = \frac{12}{13}$

(5.6) y_2 $-5/13$

(5.2) $\cos(\theta)$ $\frac{12}{13}$

(5.7) $\sin(\pi - \theta)$ $= -\sin(-\theta) = \sin(\theta) = y_1 = 5/13$

(5.3) y_1 $5/13$

(5.8) $\sin(3\pi - \theta)$ $= \sin(2\pi + \pi - \theta) = \sin(\pi - \theta) = 5/13$

(5.4) $\sin(\theta)$ $5/13$

(5.9) $\sec(-\theta)$ $= \frac{1}{\cos(-\theta)} = \frac{1}{\cos(\theta)} = \frac{13}{12}$

(5.5) $\tan(\theta)$ $= \frac{\sin\theta}{\cos\theta} = \frac{5/13}{12/13} = 5/12$

(5.10) $\csc(\pi - \theta)$ $= \frac{1}{\sin(\pi - \theta)} = 13/5.$