Question 2:

Evaluate the following expression. Answer in the simplest form. (2 points each)

(2.1) $\log_5(25\sqrt{5})$

(2.2) $\log \frac{1}{10000}$

(2.3) $81^{\frac{3}{4}}$

(2.4) $\log_8 2^{6.3}$

(2.5) $\left(\frac{3}{2}\right)^{-3}$

Question 4:

Let x, y, v, w be real numbers such that

$$\log_4 x = 1.35, \qquad \log_4 y = -0.2,$$
$$\ln v = 2.8, \qquad \ln w = 0.4.$$

Evaluate the following (2 points each).

(4.1) $\log_4 \left(\frac{xy}{4}\right)$

(4.2) $\log_4(x^{100})$

(4.3) $\log_w v$

(4.4) $\log_{16} y$

(4.5) $\frac{v}{w^2}$

Question 5:
Let f be the function defined by

$$f(x) = 3 \cdot e^{5x}.$$

(5.1) (5 points) Find a formula for f^{-1}.

(5.2) (5 points) Let g be the function defined by

$$g(x) = \ln f(x).$$

The graph of g is a line. Find the slope and the y-intercept of the graph of g.

Question 6:

A colony of bacteria is growing exponentially, doubling in size every 100 minutes. At time $t = 0$, the number of bacteria is 300.

(6.1) (5 points) Find the function f such that $f(t)$ is equal to the number of bacteria at time t, where the unit of t is minutes.

(6.2) (5 points) For this problem, use the approximations $\log 2 \approx 0.3$ and $\log 3 \approx 0.5$. Approximately how many minutes will it take for the number of bacteria to be 1800?

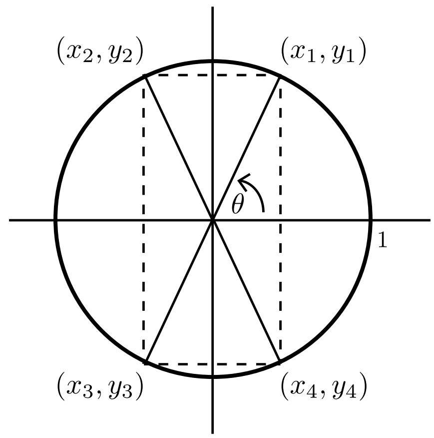
(5) (a) Find the length of a side of a right triangle that has a hypotenuse of length 2 and an angle of $\frac{\pi}{4}$ radians.

(b) Find the area of a 1 radian slice in a circle with radius 5.

(c) Calculate $\sin\frac{\pi}{12}$ given that $\cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$. Please simplify your answer.
hint: $8 - 4\sqrt{3} = \left(\sqrt{6} - \sqrt{2}\right)^2$

Question 5:

The following figure is the unit circle centered at the origin. The angle between the positive x-axis and the line passing through the origin and the point (x_1, y_1) is equal to θ. The line passing through (x_1, y_1) and (x_2, y_2) and the line passing through (x_3, y_3) and (x_4, y_4) are parallel to the x-axis. The line passing through (x_2, y_2) and (x_3, y_3) and the line passing through (x_4, y_4) and (x_1, y_1) are parallel to the y-axis.



Given that $x_4 = \frac{12}{13}$. Evaluate the following. You do not need to show work. (1 point each)

(5.1) x_1

(5.2) $\cos(\theta)$

(5.3) y_1

(5.4) $\sin(\theta)$

(5.5) $\tan(\theta)$

(5.6) y_2

(5.7) $\sin(\pi - \theta)$

(5.8) $\sin(3\pi - \theta)$

(5.9) $\sec(-\theta)$

(5.10) $\csc(\pi - \theta)$