

UC Berkeley
Department of Mathematics
Math 32– Midterm 2 Jeff Hicks

Name:

UID:

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- Please do not turn over this page until instructed to do so.
 - This exam contains 7 problems, of which we will score 6 problems. Indicate to us which 6 problems you would like us to grade by checking the small box at the top of the page. We will only grade 6 problems. Each problem is worth 10 points, for a total score of 60 points on this exam.
 - There are no notes or calculators allowed during the examination.
 - Should you finish during the last 15 minutes of the exam period, please remain seated *until we have collected all of the exams* as other students will still be working.
 - Solutions without work shown may not receive full credit. Box the solution you would like us to grade on each problem.
 - This exam contains 8 pages (including this page.)

1. Using that constants x and y satisfy $\log_2(x) = \frac{1}{2}$, and $\log_2(y) = \frac{1}{3}$, simplify each of the following expressions to a number.

(a) (1 point) $\log_2(16)$

$$2 \cdot 2 \cdot 2 \cdot 2 = 16$$
$$\log_2(16) = 4$$

(b) (2 points) $\log_2(x \cdot y)$

$$\log_2(x \cdot y) = \log_2(x) + \log_2(y)$$
$$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

(c) (3 points) $\log_2(x^3)$

$$\log_2(x^3) = 3 \cdot \log_2(x) = 3 \cdot \frac{1}{2} = \frac{3}{2}$$

(d) (4 points) $\log_2\left(\frac{4x}{y}\right)$.

$$\log_2\left(\frac{4x}{y}\right) = \log_2(4x) - \log_2(y)$$
$$= \log_2(4) + \log_2(x) - \log_2(y)$$
$$= 2 + \frac{1}{2} - \frac{1}{3} = \frac{13}{6}$$

2. (a) (2 points) Calculate $\ln(e^2)$.

$$\ln(e^2) = 2 \ln(e) = 2$$

- (b) (4 points) Estimate $5 \cdot e^{0.002}$.

$$e^{0.002} \approx 1 + .002$$

$$5 \cdot 1.002 = 5.01$$

- (c) (4 points) Estimate $\ln(.992)$.

$$\ln(x) \approx x - 1$$

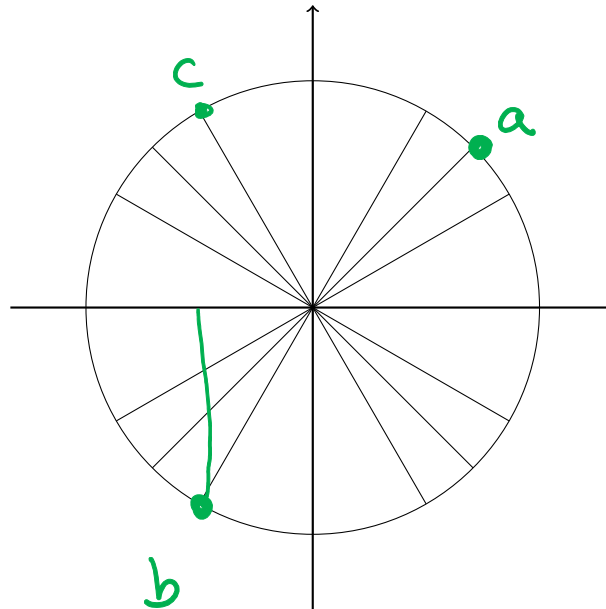
$$\approx .992 - 1 = -.008$$

3. Mark the location of the following angles on the unit circle. All measures are in radians.

(a) (1 point) $\theta = \pi/4$

(b) (2 points) $\theta = -\frac{2\pi}{3}$

(c) (2 points) $\theta = \frac{8\pi}{3}$. $4\pi + \frac{2\pi}{3}$



(a) (2 points) What is the y -coordinate of the angle marked in part b.

$$-\frac{\sqrt{3}}{2}$$

(b) (3 points) What are the values of $\cos\left(-\frac{\pi}{4}\right)$ and $\tan\left(-\frac{\pi}{4}\right)$.

$$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\tan\left(-\frac{\pi}{4}\right) = -1$$

4. Solving for Tan and Cos from Sin

(a) (5 points) Suppose that $\sin(\theta) = \frac{1}{3}$ and $0 \leq \theta \leq \pi/2$. Compute $\tan(\theta)$.

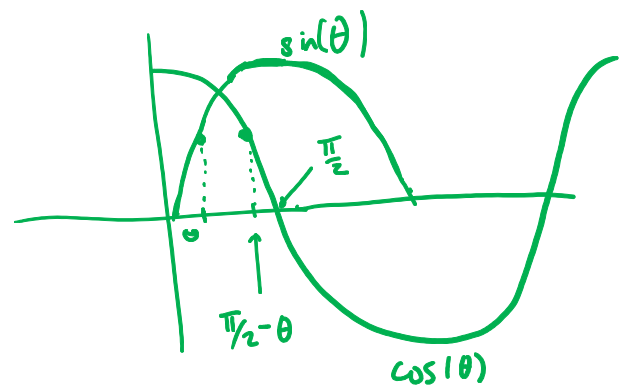
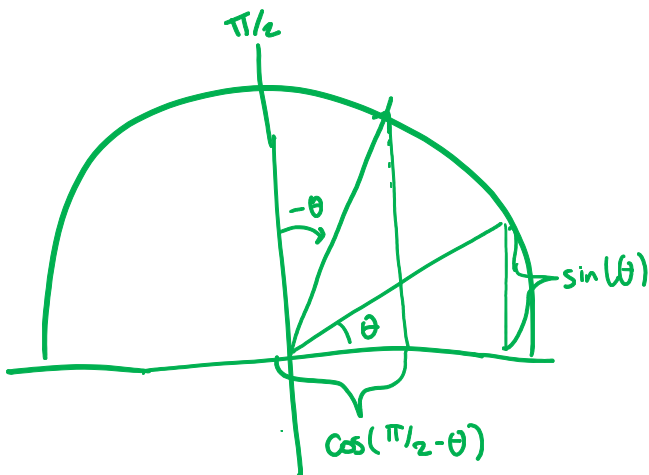
$$\begin{array}{l|l|l} \sin(\theta) = \frac{1}{3} & (\cos \theta)^2 = \frac{8}{9} & \tan \theta = \frac{\sin \theta}{\cos \theta} \\ (\cos \theta)^2 + (\sin \theta)^2 = 1 & \cos \theta = \pm \sqrt{8/9} = \pm 2\sqrt{2}/3 & = \frac{1/3}{2\sqrt{2}/3} = \frac{1}{2\sqrt{2}} \\ (\cos \theta)^2 + \frac{1}{9} = 1 & \text{Since angle is in Quad I, + value.} & \end{array}$$

(b) (2 points) What is $\sin(\theta) - \sin(\theta + 4\pi)$?

$$\begin{aligned} & \sin(\theta) - \sin(\theta + 4\pi) \\ &= \sin(\theta) - \sin(\theta) \\ &= 0 \end{aligned}$$

(c) (3 points) Suppose as before that $\sin(\theta) = \frac{1}{3}$. What is $\cos(\frac{\pi}{2} - \theta)$?

$$\boxed{\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta) = \frac{1}{3}}$$



5. Let P be a principal investment of \$ 400 dollars, invested at the start of 2019 with a 5 percent interest rate.
- (a) (2 points) Calculate how much the investment will accrue after 2 years if the interest is not compounded.

$$\begin{aligned}
 A(t) &= P \cdot (1 + rt) \\
 &= 400 \cdot (1 + .05 \cdot 2) \\
 &= 400 \cdot (1.1) = \boxed{440}
 \end{aligned}$$

- (b) (5 points) Calculate how much the investment will accrue after 2 years if the interest is compounded yearly.

$$\begin{aligned}
 400 \cdot (1.05) &= 420 && \text{After year 1} \\
 420 \cdot (1.05) &= \boxed{441.00} && \text{After year 2}
 \end{aligned}$$

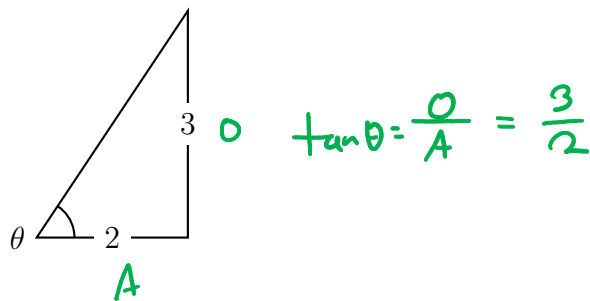
- (c) (3 points) Estimate how long it will take to become \$ 800 if continuously compounded at 5 percent interest.

Rule of 70: Doubling time is about $\frac{70}{5\%} = 14$ years

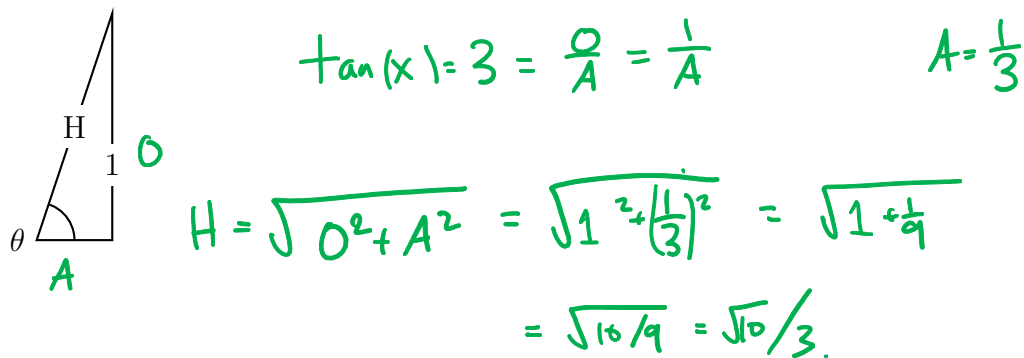
400 \rightarrow 800 1 doubling

$\boxed{\sim 14 \text{ years}}$

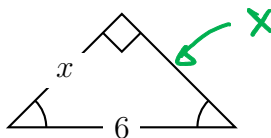
6. (a) (2 points) Compute $\tan(\theta)$ for the triangle drawn below.



- (b) (5 points) Given that $\tan(x) = 3$, find the length H .



- (c) (3 points) On this isosceles right triangle, determine the length of the side x .



$$\sqrt{x^2 + x^2} = 6$$

$$x \cdot \sqrt{2} = 6$$

$$x = \frac{6}{\sqrt{2}} = 6 \frac{\sqrt{2}}{2} = 3\sqrt{2}$$

7. (10 points) Given that $\log_3(x) = 2.5$, find the base b so that $\log_b(x) = 5$.

$$\log_3(x) = 2.5$$

$$5 = \log_b(x) = \frac{\log_3(x)}{\log_3(b)}$$

$$5 = \frac{2.5}{\log_3(b)}$$

$$\log_3(b) = \frac{2.5}{5} = \frac{1}{2}$$

$$3^{\log_3(b)} = 3^{\frac{1}{2}}$$

$$b = 3^{\frac{1}{2}} = \sqrt{3}.$$

Good Luck!