Math 32: Precalculus. Spring 2018
Midterm 1
Thunwa "Nics" Theerakarn

Name: Solution ____________________ SID: ______________

Discussion (pick one): 101. 10-11a. Katie Henderson
102. 11-12p. Katie Henderson
103. 1-2p. Chris Gerig
104. 2-3p. Chris Gerig

- There are 5 problems. Each problem is worth 10 points, for a total of 50 points.
- The total time is 50 minutes.
- Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back and clearly indicate that you did so.
- Unless otherwise specified, you must show and justify your work in logical steps.
- Write legibly. Cross out the parts that you don't want them to be graded.
- This exam is closed book: no note, no book, no calculator or any electronics device.
- I will strictly follow all University and Math Department academic Honesty Policies
- In case of an emergency alarm, leave the exam on the desk and walk out. You may or may not be allowed back to complete the work so please do not discuss it with other students.

1	
2	
3	
4	
5	
Total	

Question 1:

Find the set of all real numbers x such that

$$\frac{2x-1}{5-x} \geq 1.$$

Case $5-x>0$ (i.e. $x<5$)

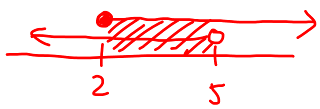
$$\frac{2x-1}{5-x} \geq 1$$

$$\Leftrightarrow \quad 2x-1 \geq 5-x$$

$$3x \geq 6$$

$$x \geq 2$$

We have that $x \geq 2$ and $x<5$.



In this case, the set of solution is $\{x \mid 2 \leq x < 5\} = [2,5)$.

Case $5-x<0$ (i.e. $x>5$)

$$\frac{2x-1}{5-x} \geq 1$$

$$\Leftrightarrow \quad 2x-1 \leq 5-x$$

$$3x \leq 6$$

$$x \leq 2$$

We have that $x \leq 2$ and $x>5$

There is no such x.

Therefore, the set of all x s.t. $\frac{2x-1}{5-x} \geq 1$ is the interval $[2,5)$.

Question 2:

(2.1) (5 points) Let L be the line in the xy-plane that contains the points $(-1, 7)$ and $(2, 4)$. Find the equation of the line in the xy-plane that is parallel to L and contains the point $(0, 6)$. Simplify the equation.

The slope of L is $\frac{4-7}{2-(-1)} = \frac{-3}{3} = -1$.

Let $y = mx+b$ be the equation of the line we want.

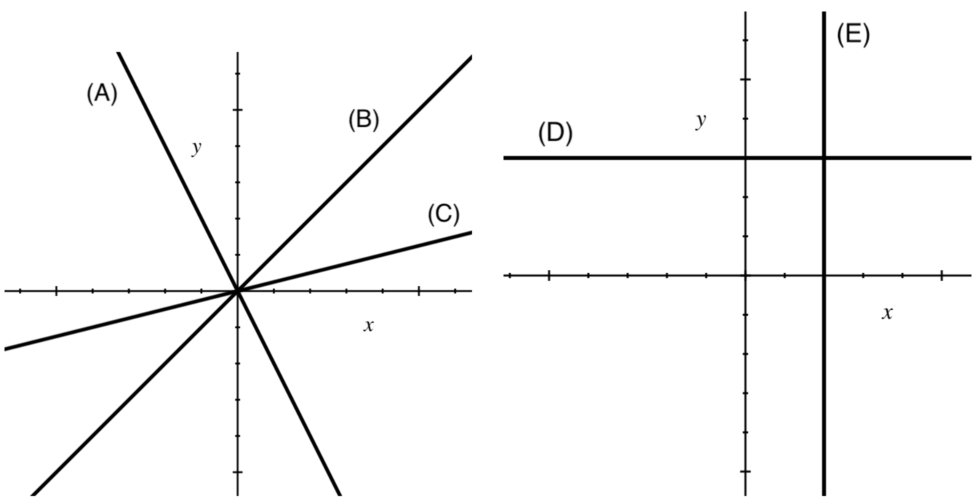
The line is parallel to L $\Rightarrow$ $m = -1$.

We have $y = (-1)\cdot x + b$.

The line passes thru $(0,6)$. We must have that $6 = (-1)\cdot 0 + b$

That is $b = 6$. The equation we want is $\boxed{y = (-1)\cdot x + 6}$.

(2.2) (5 points) Inferring from the following pictures, match each line with its equation. You do not need to show any work.



D ____ $y = 3$ horizontal line

B ____ $y = x$

C ____ $y = \frac{1}{4}x$

$\}$ slope $=1$ steeper than slope $= \frac{1}{4}$.

A ____ $y = -3x$ negative slope

E ____ $x = 2$ vertical line

Question 3:

Let f and g be the functions defined by

$$f(x) = \frac{1}{|2x-1|} \quad \text{and} \quad g(x) = \sqrt{x-1}.$$

(3.1) (2 points) Evaluate $(g \circ f)(0)$.

$(g \circ f)(0) = g(f(0))$

$f(0) = \frac{1}{|2 \cdot 0 - 1|} = \frac{1}{|-1|} = \frac{1}{1} = 1.$ $\quad g(1) = \sqrt{1-1} = 0.$

(3.2) (2 points) Find the domain of f.

$f(x)$ is defined everywhere except when $|2x-1| = 0$.
That is when $x = \frac{1}{2}$.

Domain$(f) = \{x \mid x \neq \frac{1}{2}\} = (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$.

(3.3) (2 points) Find the domain of g.

$g(x)$ is defined if and only if $(x-1) \geq 0$. That is $x \geq 1$.

Domain$(g) = \{x \mid x \geq 1\} = [1, \infty)$

(3.4) (4 points) Find the domain of $g \circ f$.

$x \neq \frac{1}{2}$

Domain$(g \circ f) = \{x \mid (x$ is in domain of $f)$ and $(f(x)$ is in domain of $g)\}$.

$f(x)$ is in Domain(g) if and only if $f(x) \geq 1$.

$$\frac{1}{|2x-1|} \geq 1$$

$$1 \geq |2x-1| \quad (\text{because } |a| \geq 0)$$

This means $\quad 2x-1 \leq 1$ and $2x-1 \geq -1$

$2x \leq 2$ and $2x \geq 0$

$x \leq 2$ and $x \geq 0$

Therefore, Domain$(g \circ f) = \{x \mid x \neq \frac{1}{2}$ and $0 \leq x \leq 2\} = [0, \frac{1}{2}) \cup (\frac{1}{2}, 2]$.

Question 4:

(4.1) (5 points) Let f be the function defined by

$$f(x) = 3x - 1.$$

Find a formula for f^{-1}.

Set $y = 3x - 1$ and solve for x in terms of y.

$$y + 1 = 3x$$

$$\frac{y+1}{3} = x.$$

Therefore, $f^{-1}(y) = \dfrac{y+1}{3}$.

↑

y is just a name of the variable.

So, we can also say $f^{-1}(x) = \dfrac{x+1}{3}$,

(4.2) (5 points) Let f be the function defined by

$$f(x) = \frac{14x + 19}{2x - 22}.$$

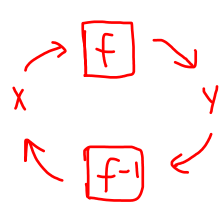
f is a one-to-one function. Compute $(f^{-1} \circ f)(10)$.

It is given that f is a 1-1 function.

Hence, f has an inverse function f^{-1}.

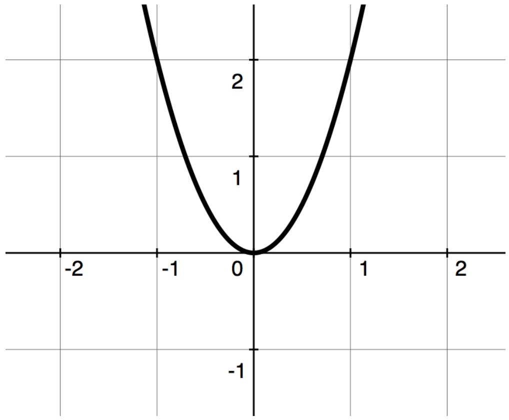
By definition, $f^{-1}(y) = x$ if and only if $f(x) = y$.

So, $f^{-1}(f(x)) = x$. We have $(f^{-1} \circ f)(10) = 10$.

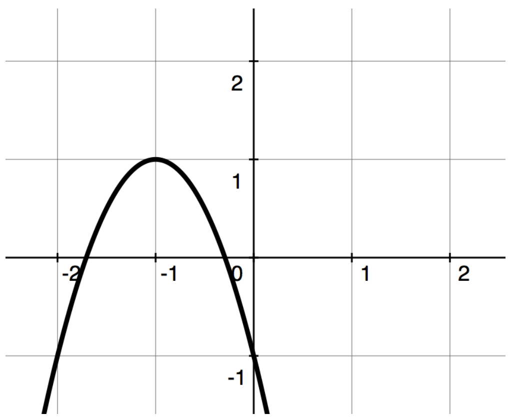


Question 5:

The following picture shows the graph of the function $f(x) = 2x^2$.



Let g be the function whose graph is the parabola in the following picture.
The domain of g is the set of all real numbers.



(5.1) (2 points) Find the value of x where $g(x)$ attains its maximum and find the maximum value of $g(x)$.

max of $g(x) = 1$ at $x = -1$.

(5.2) (2 points) Which of the following is correct?

(A) $g(x) = f(x+1) + 1$

(B) $g(x) = f(x+1) - 1$

(C) $g(x) = -f(x+1) + 1$

(D) $g(x) = -f(x+1) - 1$

(E) $g(x) = f(x-1) + 1$

(F) $g(x) = f(x-1) - 1$

(G) $g(x) = -f(x-1) + 1$

(H) $g(x) = -f(x-1) - 1$

(5.3) (3 points) Write $g(x)$ in the form

$$g(x) = ax^2 + bx + c$$

where a, b, and c are real numbers.

$f(x) = 2x^2$

$g(x) = -f(x+1) + 1$ (from 5.2)

$= -\left(2(x+1)^2\right) + 1$

$= -\left(2(x^2+2x+1)\right) + 1$

$= -2(x^2+2x+1) + 1$

$= -2x^2 - 4x - 2 + 1$

$= -2x^2 - 4x - 1.$

(5.4) (3 points) Find all real numbers x such that $g(x) = 0$.

From (5.2): $g(x) = -f(x+1) + 1$

$= -2(x+1)^2 + 1$

$g(x) = 0 \iff -2(x+1)^2 + 1 = 0$

$-2(x+1)^2 = -1$

$(x+1)^2 = \frac{-1}{-2} = \frac{1}{2}$

$x+1 = \pm\sqrt{\frac{1}{2}}$

$x = -1 \pm \sqrt{\frac{1}{2}}$

Alternatively, from (5.3),

$g(x) = -2x^2 - 4x - 1$

$g(x) = 0 \iff 0 = -2x^2 - 4x - 1$

from the quadratic formula,

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(-1)}}{2(-2)}$$

$$= \frac{4 \pm \sqrt{16-8}}{-4}$$

$$= \frac{4 \pm \sqrt{8}}{-4} = \frac{4 \pm 2\sqrt{2}}{-4}$$

$$= -1 \pm \frac{\sqrt{2}}{2} = -1 \pm \frac{1}{\sqrt{2}}.$$

Extra space.