UC Berkeley
Department of Mathematics
Math 32– Midterm 2 Jeff Hicks

Name:

UID:

- Please do not turn over this page until instructed to do so.
- This exam contains 7 problems, of which we will score 6 problems. Indicate to us which 6 problems you would like us to grade by checking the small box at the top of the page. We will only grade 6 problems. Each problem is worth 10 points, for a total score of 60 points on this exam.
- There are no notes or calculators allowed during the examination.
- Should you finish during the last 15 minutes of the exam period, please remain seated *until we have collected all of the exams* as other students will still be working.
- Solutions without work shown may not receive full credit. Box the solution you would like us to grade on each problem.
- Should you need extra space for work, you may use the empty pages included at the end of the exam. Mark clearly that your work is contained on a separate page in the original problem sheet.
- This exam contains 12 pages (including this page.)

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1. Compute the following values.

   (a) (1 point) $\log_2(32)$

   $\log_2(2^5) = 5$

   (b) (2 points) $\log_3\left(\frac{1}{27}\right)$

   $\log_3(3^{-3}) = -3$

   (c) (3 points) $\log_{16}(4)$

   $\log_{16}(\sqrt{16}) = \log_{16} 4^{1/2} = \frac{1}{2}$

   (d) (4 points) Using the approximations

   $$\log_{10}(5) \sim 0.698$$
   $$\log_{10}(7) \sim 0.845,$$

   approximate $\log_{10}(35)$ and $\log_{10}(50)$.

   $\log_{10}(35) = \operatorname{Log}_{10}(7 \cdot 5)$

   $= \operatorname{Log}_{10}(7) + \operatorname{Log}_{10}(5) = .698 + .845$

   $= 1.543$

   $\log_{10}(50) = \log_{10}(5 \cdot 10) = \log_{10}(5) + \log_{10}(10)$

   $= 1.698$

2. Let $f(x) = \ln(x)$ and let $g(x) = e^{2x-1}$.

(a) (2 points) Compute $(f \circ g)(3)$.

$$\ln(e^{2x-1}) = 2x-1$$

$$2 \cdot 3 - 1 = 5$$

(b) (4 points) Estimate $g(0.51)$.

$$e^{2 \cdot .51 - 1} = e^{1.02-1} = e^{.02} \approx 1 + .02 = 1.02.$$

(c) (4 points) Write down an inverse function $g^{-1}(y)$ for $g(x)$.

$$y = e^{2x-1}$$

$$\ln(y) = \ln(e^{2x-1})$$

$$\ln(y) = 2x-1$$

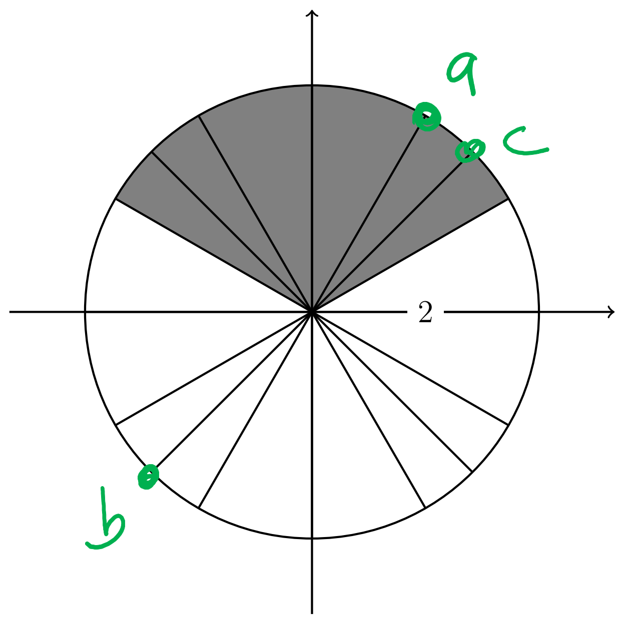
$$\frac{\ln(y)+1}{2} = x$$

3. Mark the location of the following angles on the following circle. The markings on the circle are at multiples of $\pi/6$ and $\pi/4$ radians. All measures are in radians.

   (a) (1 point) $\theta = \pi/3$

   (b) (1 point) $\theta = -\frac{3\pi}{4}$

   (c) (2 points) $\theta = \frac{9\pi}{4}$.



   (a) (2 points) What are the values of $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$?

$\sin\left(\frac{\pi}{3}\right) = \sqrt{3}/2$

$\cos\left(\frac{\pi}{3}\right) = 1/2$

   (b) (4 points) What is the area of the slice marked in gray. (Note: the radius of the circle is two.)

$\Theta = \frac{2\pi}{3}$

$\frac{1}{2} r^2 \Theta = \frac{1}{2} 2^2 \cdot \frac{2\pi}{3}$

$= 4\pi/3$

4. (a) (4 points) Suppose that $\sin(\theta) = \frac{1}{3}$ and $0 \le \theta \le \pi/2$. Compute $\tan(\theta)$.

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$= \pm\sqrt{1-\frac{1}{9}} = \pm\sqrt{8/9} = +2\sqrt{2}/3$

θ in Quadrant I

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/3}{2\sqrt{2}/3} = \frac{1}{2\sqrt{2}}$

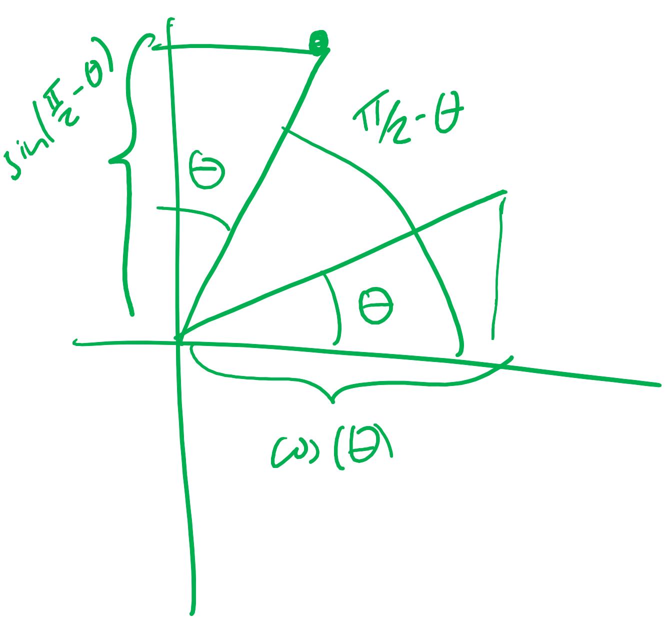
(b) (2 points) Let $\sin(\theta) = \frac{1}{3}$ as before. What is $\cos(\theta) - \cos(-\theta)$?

$\cos(\theta) - \cos(-\theta) = \cos\theta - \cos(\theta) = 0.$

(c) (4 points) Let $\sin(\theta) = \frac{1}{3}$ as before. What is $\sin\left(\frac{\pi}{2} - \theta\right)$?

$\sin(\theta) = \frac{1}{3}$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta) = 2\sqrt{2}/3$



5. On January 1st 2009, I made an investment of 200 dollars. After 10 years of earning continuously compounded interest, the value of the investment has doubled to 400 dollars by 2019.

   (a) (2 points) What year will it be when the investment is worth 800 dollars (provided that the investment keeps the same rate of return.)

$$200 \xrightarrow{} 400 \xrightarrow{} 800$$

10 years to double 1 — 2009

10 years to double 2x — 2019

$$\boxed{2029}$$

   (b) (3 points) Estimate the interest rate at which the investment is continuously compounded.

"Rule of 70"

$$\frac{70}{R} \approx 10 \qquad R = 7$$

   (c) (5 points) Using your answer from part (b), estimate how much the account was worth on February 7th, 2009. *Hint: February 7th, 2009 is 36 days after the initial investment, or approximately $\frac{1}{10}$th of a year after the initial deposit.* [1]

$$200 \cdot e^{\frac{1}{10} \cdot .07}$$

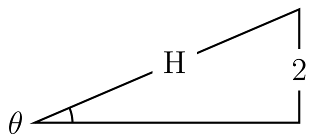
$$= 200 \cdot e^{.007}$$

$$\approx 200(1 + .007)$$

$$= 200(1.007) = 201.40$$

[1] If you were not able to solve part (b), please state that you were not able to work out part (b), and proceed working through the problem with the assumption that the interest rate is 1% per year.

6. (a) (2 points) Given that $\sin(\theta) = \frac{1}{3}$, find the length H.

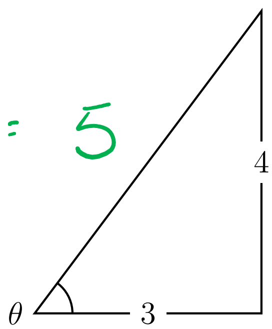


$$\sin(\theta) = \frac{O}{H} = \frac{2}{H} = \frac{1}{3}$$

$$H = 6$$

(b) (3 points) Compute $\sin(\theta)$ for the triangle drawn below.

$\sqrt{3^2+4^2} = 5$

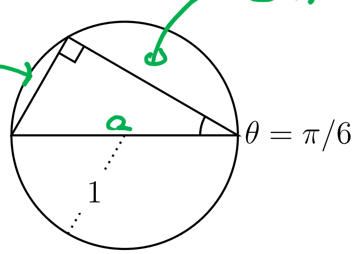


$$\sin(\theta) = \frac{4}{5}$$

(c) (4 points) Find the area of the right triangle which is inscribed in the unit circle whose corner measures $\frac{\pi}{6}$ radians .

$2\cos\theta$

$2\sin\theta$



$$\text{Area} = \frac{1}{2} O \cdot A$$

$$= \frac{1}{2} \cdot 1 \cdot \sqrt{3}$$

$$= \sqrt{3}/2$$

$$2\sin\frac{\pi}{6} = 2 \cdot \frac{1}{2} = 1$$

$$2\cos\frac{\pi}{6} = 2\frac{\sqrt{3}}{2} = \sqrt{3}$$

7. (10 points) Compute $\log_{3^x}(x)$ when $x = 27$.

$$\log_{3^x}(x) = \frac{\log_3(x)}{\log_3(3^x)} = \frac{\log_3(x)}{x}$$

when $x = 27$

$$= \frac{\log_3 27}{27} = 3/27 = 1/9.$$

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