

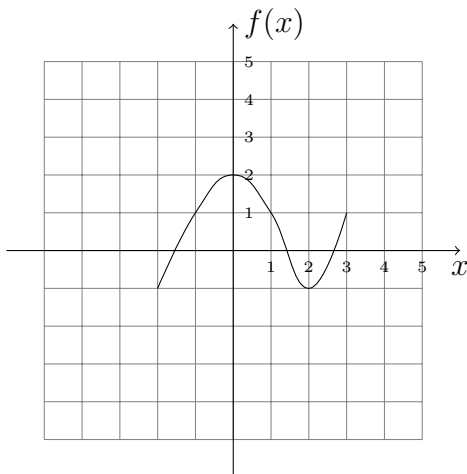
UC Berkeley
Department of Mathematics
Math 32– Midterm 1 Jeff Hicks

Name:

UID:

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- Please do not turn over this page until instructed to do so.
 - This exam contains 7 problems, of which we will score 6 problems. Indicate to us which 6 problems you would like us to grade by checking the small box at the top of the page. We will only grade 6 problems. Each problem is worth 10 points, for a total score of 60 points on this exam.
 - There are no notes or calculators allowed during the examination.
 - Should you finish during the last 15 minutes of the exam period, please remain seated *until we have collected all of the exams* as other students will still be working.
 - Solutions without work shown may not receive full credit. Box the solution you would like us to grade on each problem.
 - This exam contains 8 pages (including this page.)

1. Let $f(x)$ be the function whose graph is drawn below.



(a) (1 point) What is $f(2)$?

$$f(2) = -1$$

(b) (2 points) Explain whether or not the function $f(x)$ is one-to-one.

$$\text{No, as } f(1) = f(-1) = 1.$$

(a) (2 points) Describe the domain and range of f in interval form.

$$\text{Range: } [-1, 2]$$

$$\text{Domain: } [-2, 3]$$

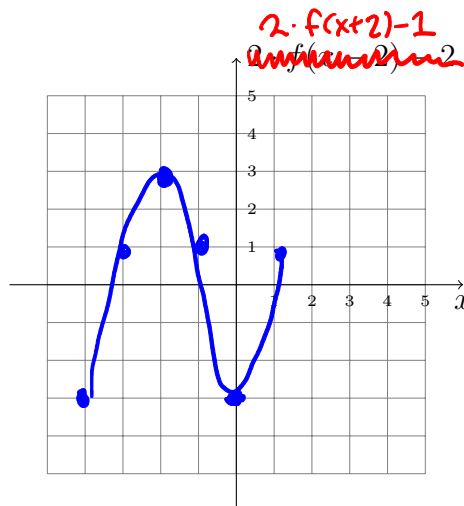
(c) (5 points) Draw the graph of the function $2 \cdot f(x+2) - 1$ in the space below.

$$2 \cdot f(x+2) - 1$$

→ shifted 2 left

→ scaled vert 2

→ shifted down 1



2. Consider the functions given by the tables below.

x	f(x)
1	4
2	2
3	1
4	5
5	3

x	g(x)
1	4
2	4
3	2
4	2
5	3

(a) (1 point) What is $f(3)$?

$$f(3) = 1$$

(b) (2 points) Compute $(f \cdot g)(5)$.

$$f(5) = 3 \quad (f \cdot g)(5) = 9$$

$$g(5) = 3$$

(c) (3 points) Compute $(g \circ f)(3)$.

$$g(f(3)) = g(1) = 4$$

(d) (4 points) Compute $(f^{-1} \circ g)(2)$.

$$f^{-1}(g(2)) = f^{-1}(4) = 1.$$

3. (a) (5 points) Give the equation of a line containing the points $(4, -2)$ and $(2, -1)$.

$$m = \frac{-2 - (-1)}{4 - 2} = -\frac{1}{2}$$

$$(y - 4) = -\frac{1}{2}(x - 4)$$

$$\boxed{y = -\frac{1}{2}x + 2}$$

- (b) (3 points) Find the equation of a line which passes through the origin and is perpendicular to the line from Part (a).

$$\text{slope} = \frac{1}{-m} = 2$$

$$y = 2x$$

- (c) (2 points) Find the point of intersection between the lines from Parts (a) and (b).

$$\begin{array}{ll} y = 2x & l_1: 2 \\ y = -\frac{1}{2}x & l_2: 1 \end{array}$$

$$\begin{array}{ll} 2x = -\frac{1}{2}x \Rightarrow & x = 0 \\ y = 2x \Rightarrow & y = 0 \end{array}$$

$$(0, 0)$$

4. (10 points) Describe the set of real numbers satisfying the inequality $|3x - 3| < x$ using interval notation.

$$|3x - 3| < x$$

or

If $3x - 3 \leq 0$
i.e. $x \leq 1$

$$-(3x - 3) < x$$
$$3 < 4x$$
$$\frac{3}{4} < x \leq 1$$

If $3x - 3 > 0$
i.e. $x > 1$

$$(3x - 3) < x$$
$$2x < 3$$
$$1 < x < \frac{3}{2}$$

$$\frac{3}{4} < x < \frac{3}{2}$$
$$\left(-\frac{3}{4}, \frac{3}{2}\right)$$

5. Consider the function

$$f(x) = \frac{x+2}{x-3}$$

(a) (2 points) Write the domain of $f(x)$ using interval notation.

Domain: Does not include 3 as we would then divide by 0
 $(-\infty, 3) \cup (3, \infty)$.

(b) (5 points) Write an equation for the function $f^{-1}(x)$.

$$y = \frac{x+2}{x-3}$$

$$y(x-3) = x+2$$

$$yx - 3y = x+2$$

$$yx - x = 2+3y$$

$$x(y-1) = 2+3y$$

$$x = \frac{3y+2}{y-1}$$

$$f^{-1}(y) = \frac{3y+2}{y-1}$$

(c) (3 points) Write down the domain and range of $f^{-1}(x)$ using interval notation.

Domain: everything but 1
 $(-\infty, 1) \cup (1, \infty)$.

Range = Domain of $f(x) = (-\infty, 3) \cup (3, \infty)$.

6. Consider the parabola given by the graph of the function

$$f(x) = -x^2 + 2x + 3.$$

(a) (2 points) Algebraically find the zeros of $f(x)$.

$$f(x) = (-x+3)(x+1)$$

$$x=3, x=-1.$$

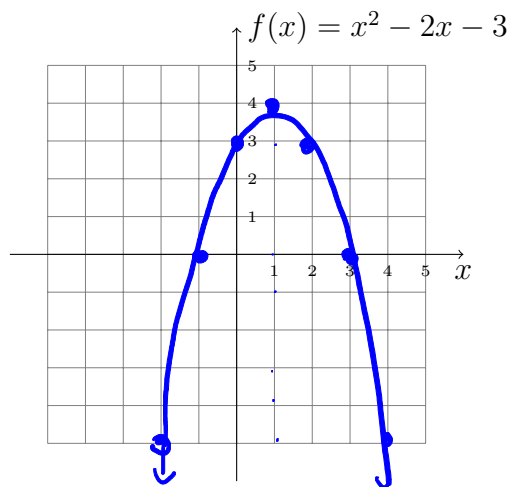
(b) (4 points) Algebraically find the vertex of $f(x)$.

$$f(x) = (-x^2 + 2x - 1) + 1 + 3$$

$$= -(x-1)^2 + 4$$

vertex is $(1, 4)$

(c) (4 points) Graph $f(x)$ below, and mark the zeros and vertex of the parabola.



7. (10 points) Find all points where the line $y = \frac{3}{4}x$ intersects the circle of radius 5 centered at the origin.

$$y = \frac{3}{4}x$$

$$x^2 + y^2 = 5^2$$

$$x^2 + \left(\frac{3}{4}x\right)^2 = 5^2$$

$$16x^2 + 9x^2 = 5^2 \cdot 16$$

$$25x^2 = 25 \cdot 16$$

$$x = \pm 4$$

$$y = \pm 3.$$

$(4, 3)$ and $(-4, -3)$